Washington University in St.Louis

Radar Imaging with Multiple Space-Time Data Cubes and Site-Specific Knowledge

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OUTLINE

A. Status Report 31 August 2004

1. Problem Formulation
2. Imaging Algorithm
3. Medium-Scale Simulation Results

B. New Results and Progress

1. Full-Scale Simulation Efforts
2. Computational Challenges
3. Computational Solutions

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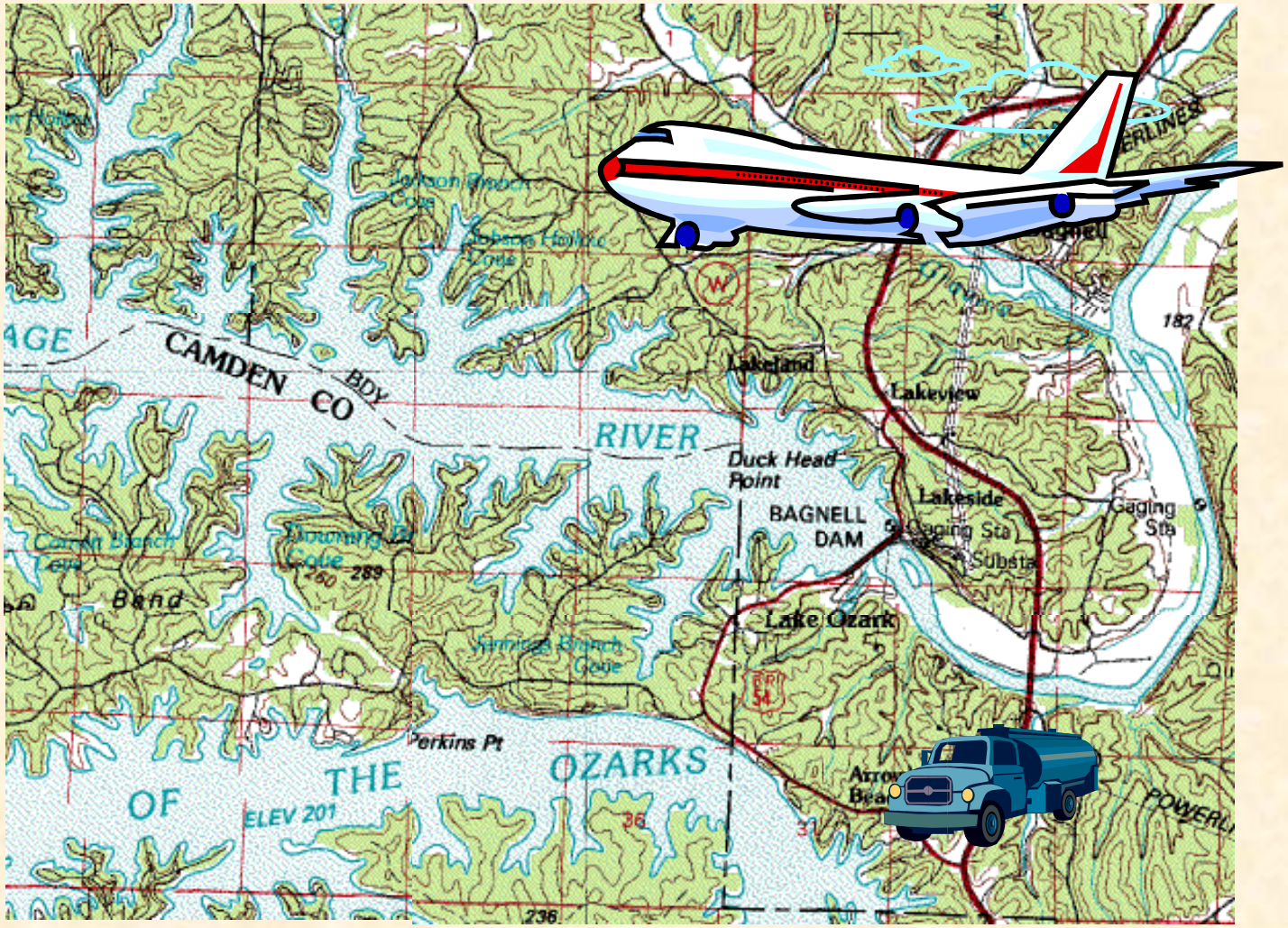
CONTEXT


CAMDEN CO
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Lakeview
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BAGNELL DAM
Lakeside
Gaging Sta
Lake Ozark
Perkins Pt
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ELEV 201
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CONTEXT

- Airborne multisensor pulse-Doppler surveillance radar
- Arbitrary flight path around region of interest
- Ground subdivided into pixels or ground patches
- Known range, and angle of each patch with respect to platform
- Known illumination pattern

Objective: Determine ground scattering function

APPROACH

- Physical modeling: constant-γ (Lambertian) scattering model (Barton, Skolnik, others)

- Data modeling: structured covariance
  - Received data modeled as 0-mean complex Gaussian vectors whose covariances are linear transformations of the scattering function Γ

- Maximum-likelihood methodology is used to estimate the unknown scattering function
  - Expectation-Maximization (EM) algorithm used to compute maximum-likelihood estimate

PROBLEM FORMULATION

- Region pixelized into *N* ground patches
  - Size of patch commensurate with radar's resolution
- Pulse waveform transmitted at instances $k=1, 2, \ldots, K$, with known illumination pattern
- Received data across the *M* sensors and *L* range gates:

$$\mathbf{z}_k = \left[z_k(1) \quad z_k(2) \quad \ldots \quad z_k(LM) \right]^{\mathrm{T}} \sim CN\left(\mathbf{0}, \mathbf{R}_k \right)$$

where

$$\mathbf{R}_k = \sum_{n=1}^{N} \mathbf{a}(n,k)\, \mathbf{a}^{\mathrm{H}}(n,k)\, \gamma_n \lambda_{nk}$$

Value of the scattering function for the nth patch

Incident energy on the nth patch at k^{th} look

Response vector for the nth patch at k^{th} look

COMPACT NOTATION

- $\mathbf{R}_k$ is block diagonal; each block corresponds to one range gate

- Compact notation $\mathbf{R}_k = \mathbf{A}_k(\mathbf{\Gamma}\mathbf{\Lambda}_k)\mathbf{A}_k^{\mathrm{H}} + \sigma^2\mathbf{I}$

where $\mathbf{A}_k = [\mathbf{a}(1,k) \cdots \mathbf{a}(N,k)]$

$$\mathbf{\Lambda}_k = \begin{bmatrix} \lambda_{1k} & & & \mathbf{0} \\ & \lambda_{2k} & & \\ & & \ddots & \\ \mathbf{0} & & & \lambda_{Nk} \end{bmatrix} \qquad \Gamma = \begin{bmatrix} \gamma_1 & & & \mathbf{0} \\ & \gamma_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \gamma_N \end{bmatrix}$$

Objective: Maximum-likelihood estimate of Γ

MAXIMUM LIKELIHOOD ESTIMATION

- Model for received data

$$f(\mathbf{z}_1, \mathbf{z}_2, \cdots \mathbf{z}_K) = \prod_{k=1}^{K} \pi^{-LM} \det(\mathbf{R}_k)^{-1} e^{-\mathbf{z}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{z}_k}$$

- Goal:

$$\hat{\Gamma}(\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_K) = \max_{\boldsymbol{\Sigma} \text{ s.t. } \boldsymbol{\Sigma} \in S} \left\{ L \overset{\Delta}{=} \sum_{k=1}^{K} -\log(\det(\mathbf{R}_k)) - \mathbf{z}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{z}_k \right\}$$

$$S = \{ \boldsymbol{\Gamma} : \gamma_n \geq 0 \quad \forall\, n = 1, 2, \cdots, N \}$$

$$\hat{\Gamma}(\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_K) = \begin{bmatrix} \hat{\gamma}_1 & & & \mathbf{0} \\ & \hat{\gamma}_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \hat{\gamma}_N \end{bmatrix}$$

Nonlinear optimization problem

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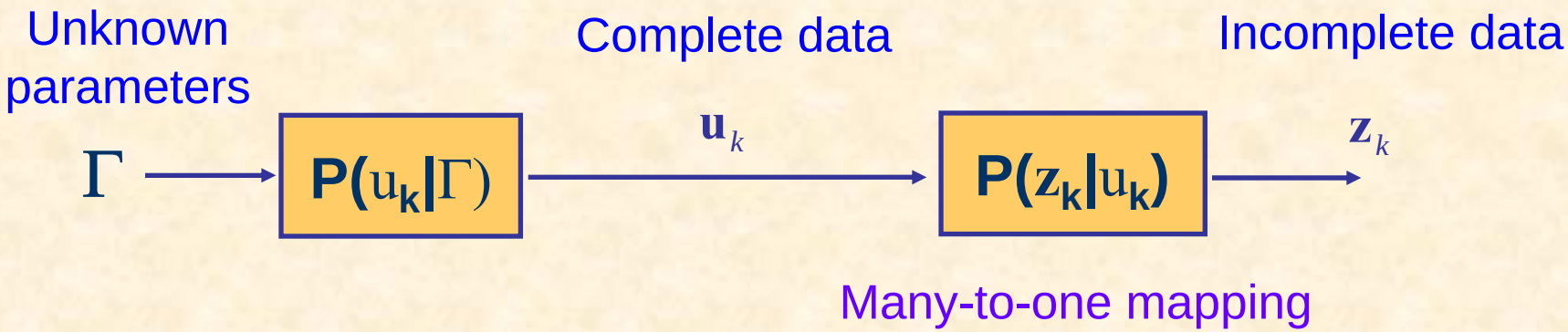
EM ALGORITHM



$$\mathbf{u}_k \sim CN(0, \mathbf{\Lambda}_k \Gamma) \qquad \mathbf{z}_k = \mathbf{A}_k \mathbf{u}_k + \mathbf{n}_k$$

- Complete-data sufficient statistic: $\dfrac{1}{K}\sum_{k=1}^{K}\dfrac{|u_{n,k}|^2}{\lambda_{nk}}$

EM ALGORITHM

- At each iteration, compute conditional expectation of complete-data sufficient statistics:

$$\hat{\gamma}_n^{(p+1)} = E\left[\frac{1}{K} \sum_{k=1}^{K} \frac{\left|u_{n,k}\right|^2}{\lambda_{nk}} \,\middle|\, \mathbf{z}_k, \hat{\mathbf{\Gamma}}^{(p)} \right]$$

$$\hat{\Gamma}^{(p+1)} = \hat{\Gamma}^{(p)} + \frac{1}{K} \sum_{k=1}^{K} \mathbf{\Lambda}_k diag\left[\hat{\Gamma}^{(p)} \mathbf{A}_k^{\mathrm{H}} \left(\mathbf{R}_k^{-1}(p) \mathbf{z}_k \mathbf{z}_k^H \mathbf{R}_k^{-1}(p) - \mathbf{R}_k^{-1}(p)\right) \mathbf{A}_k \hat{\Gamma}^{(p)}\right]$$

$$\mathbf{R}_k(p) = \mathbf{A}_k \mathbf{\Lambda}_k \hat{\Gamma}^{(p)} \mathbf{A}_k^{\mathrm{H}}$$

IMPLEMENTATION NOTES (1)

- Assume pulse compression partitions pixels into distinct range gates. Association of pixels with range gates changes is view-dependent.

- Ordered subsets (OS) [H. Hudson, R. Larkin] to accelerate convergence of EM algorithm.
  - Group incomplete data in disjoints subsets
  - Number of these sets is called *OS levels*
  - Apply standard EM algorithm for each subset at a time
  - Estimation of one subset used as starting point of other subset

- Land-use aggregation to improve the quality of the estimates
  - Average the estimated values of the scattering function that have the same land–use type.

IMPLEMENTATION NOTES (2)

- Central operation is computation of conditional mean and covariance of $\mathbf{u}_{kl}$, given data $\mathbf{z}_{kl}$ and current iterate Γ, according to model

$$\mathbf{z}_{kl} = \mathbf{A}_{kl}\mathbf{u}_{kl} + \mathbf{n}_{kl}$$

$$\mathbf{u}_{kl} \sim CN(0, \mathbf{\Lambda}_{kl}\Gamma_l)$$

where l is the range gate index.

- The matrices $\mathbf{A}_{kl}$ can be wide or long, full-rank or rank-deficient.
- Stable implementations based on the SVD of $\mathbf{A}(\Lambda\Gamma)^{1/2}$ developed.

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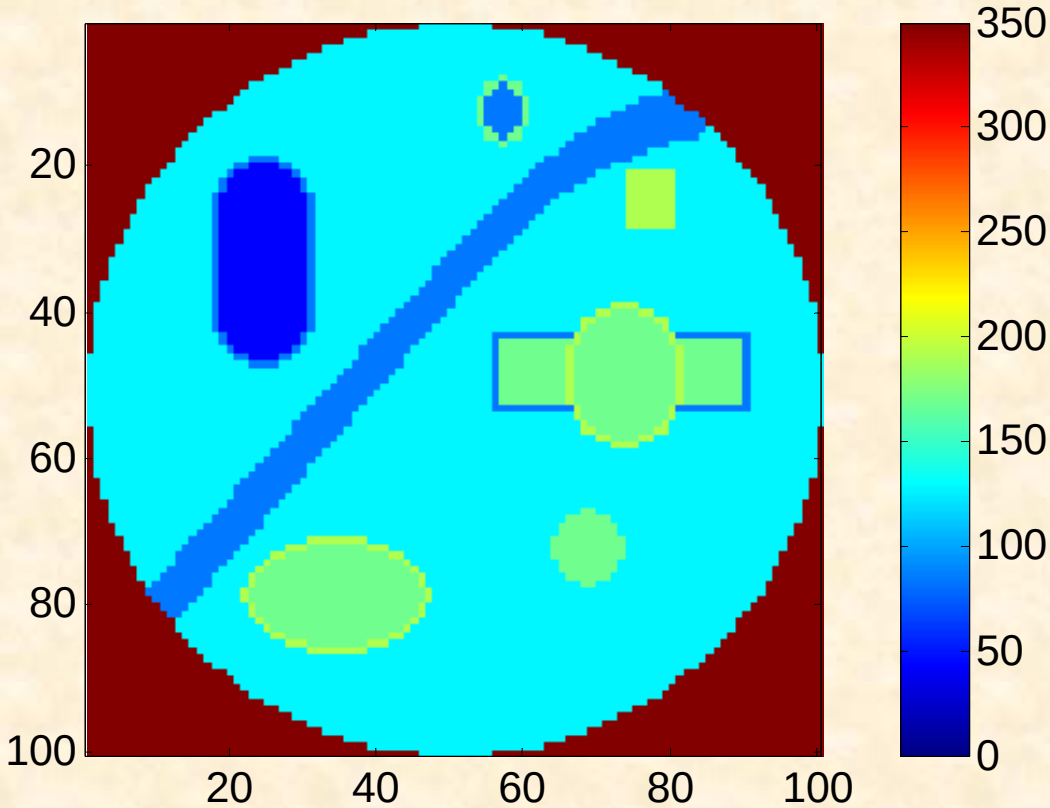
MEDIUM-SCALE SIMULATION



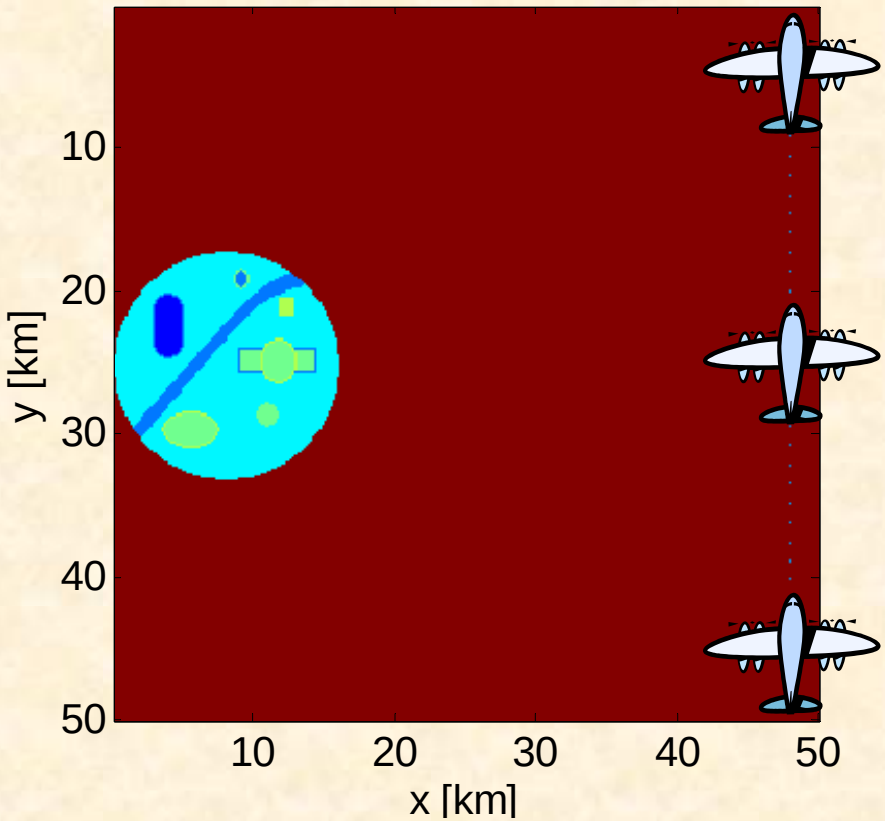
Region of interest

Circular area
16km diameter
7796 pixels
(160m on side)

Artificial scattering function

MEDIUM-SCALE SIMULATION

Ground Region of Interest and Flight Path (30 dwells)



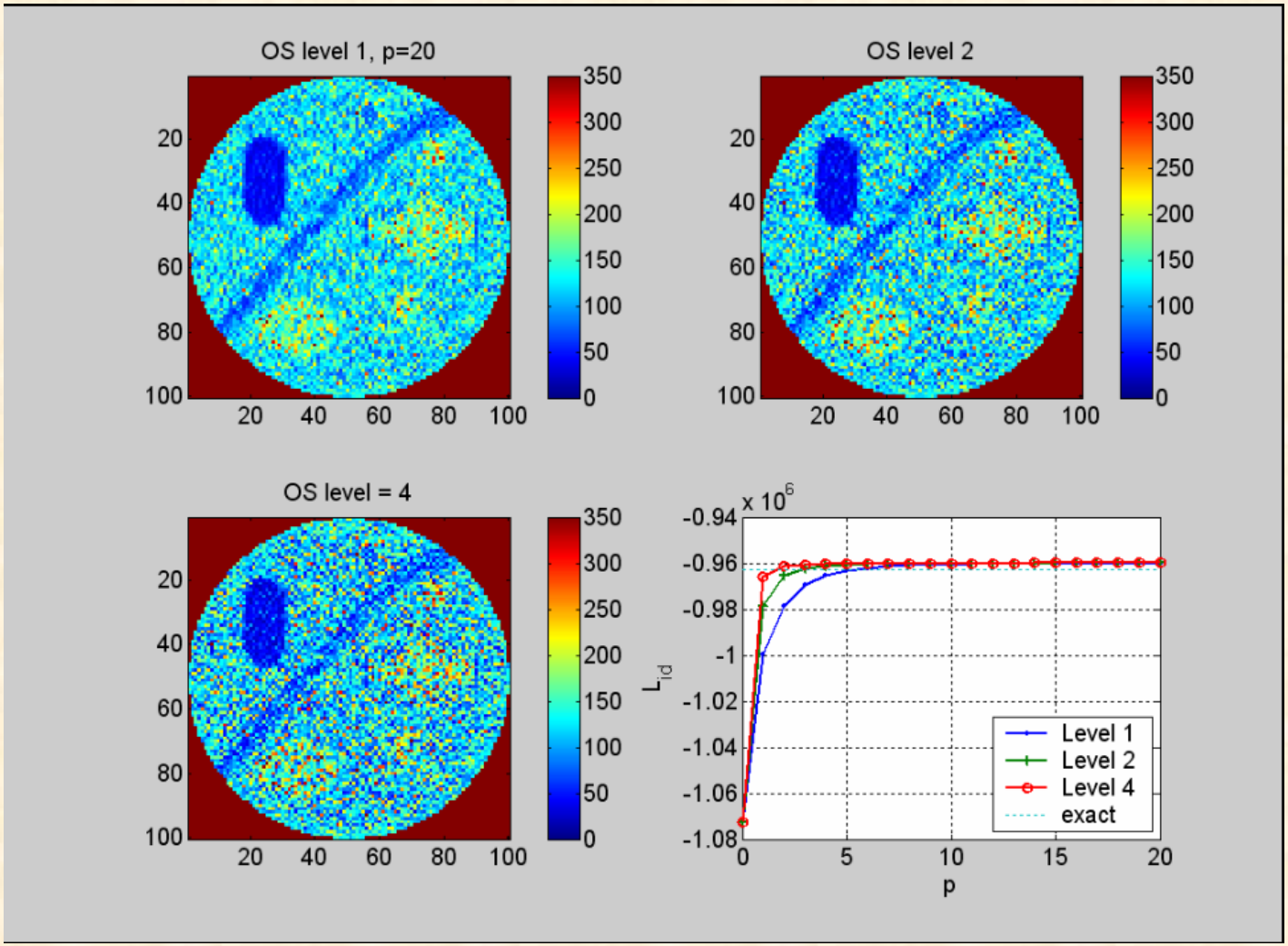
Platform Parameters

- Dist to center: 40km
- Height: 11km
- Speed: 135m/s
- Dwells: 30 (every 10s)
- 3 CPI / dwell

Radar Parameters

- Sensors: 12
- Pulses: 10
- Range gates: 100
- Frequency: 10 GHz
- Bandwidth: 10 MHz
- PRF: 2 kHz

SIMULATION RESULTS – W/O LAND USE



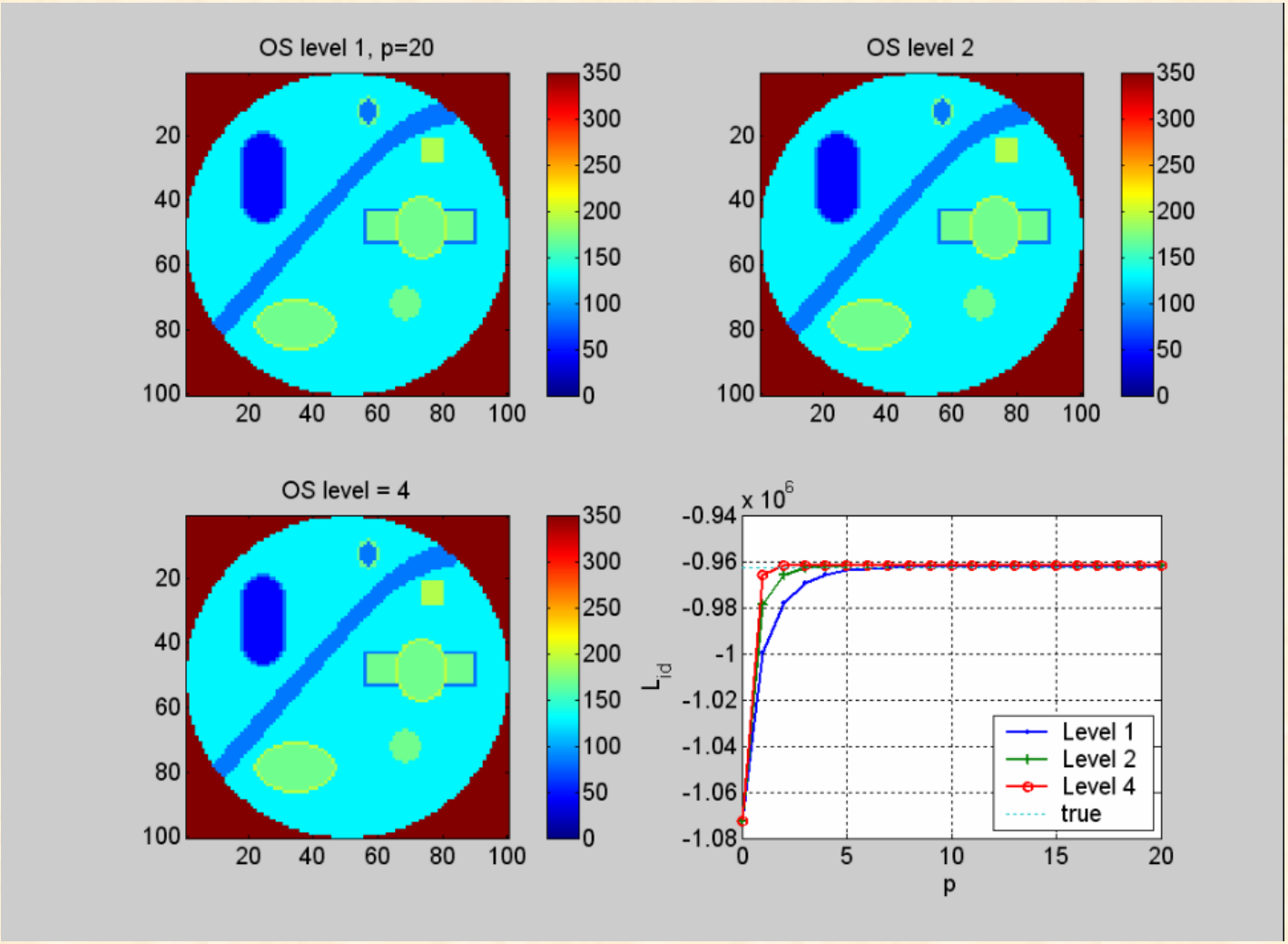
LAND-USE AGGREGATION

- In many geographical information systems, ground patches are labeled with land-use values

- Assume all pixels with the same land-use label *L* have the same scattering function g_L

- Greatly reduces the number of free parameters in the imaging problem

SIMULATION RESULTS – W/ LAND USE



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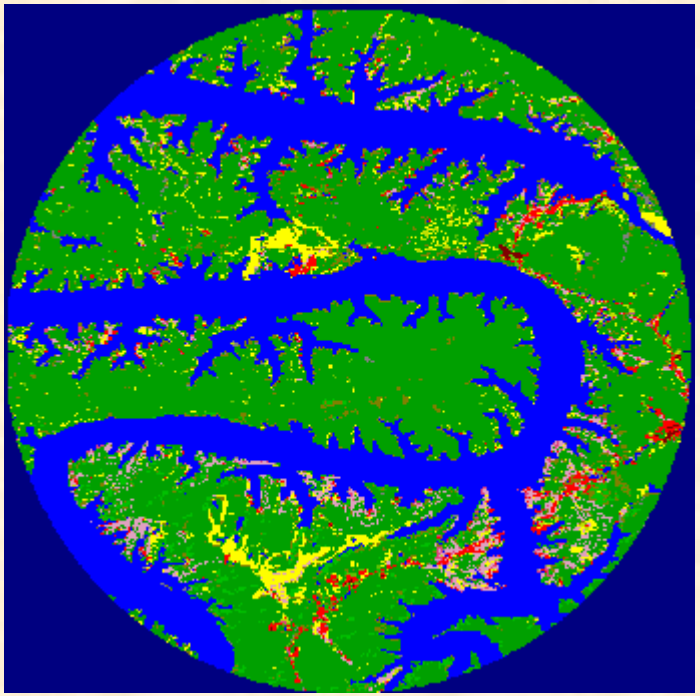
FULL-SCALE SIMULATION

- **Region of Interest**
  - Lake of the Ozarks
  - 15 km diameter
  - 197,316 pixels
  - 30m resolution



GEOGRAPHICAL DATASETS

- Obtained from USGS Seamless Data Server
  - 30m resolution
- Digital Elevation Model
  - Used for modeling geometry
- Land Use
  - Scattering function based on 21 classes of land cover
    - 9 primary classes
      - Water, Developed, Barren, Forested Upland, Shrubland, Non-Natural Woody, Herbaceous Upland Natural/Semi-natural Vegetation, Herbaceous Planted/Cultivated, Wetlands
    - Each class contains one or more categories, e.g.
      - Open Water, High-Intensity Residential, Deciduous Forest, Row Crops
  - Scattering function chosen arbitrarily for simulation

SIMULATION PARAMETERS

- **Platform**
  - Flies in circular path around region
  - Radius 25 km
  - Altitude 7 km
  - 8 different viewpoints
- **Radar**
  - f_c : 10 GHz
  - BW: 10 MHz
  - PRF: 2 KHz
  - Pulses per CPI: 38
  - ULA elements: 12
  - Range gates: 990

DATACUBE GENERATION

- Simulation model

Columns are Kronecker product of spatial and Doppler response vectors

$$\mathbf{z}_k = \mathbf{A}_k \mathbf{u}_k + \mathbf{n}_k$$

$$\mathbf{u}_k \sim CN(0, \mathbf{\Lambda}_k \Gamma)$$

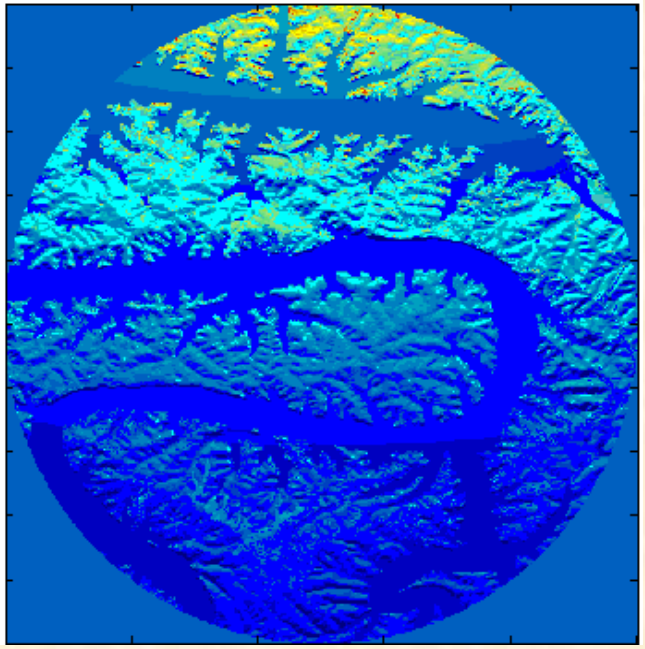
Scattering function

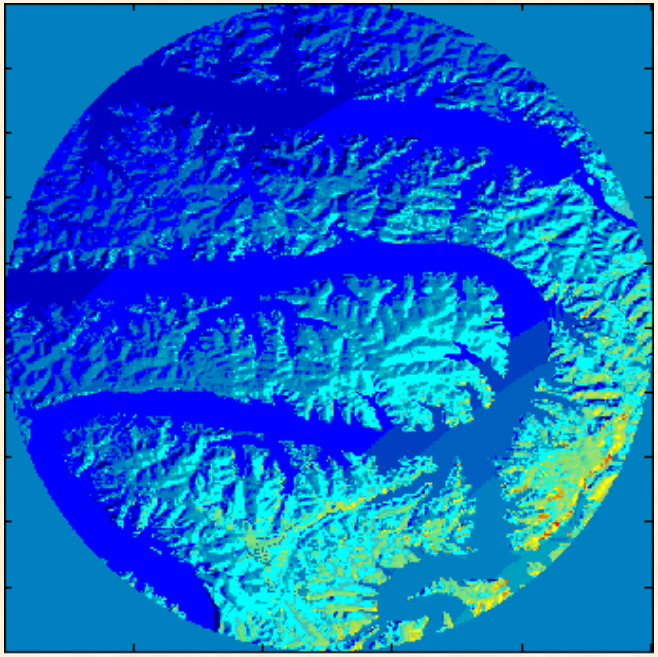
Diagonal illumination matrix for kth pulse

- Incident energy incorporates range and projected area of patch
- Patches hidden from radar are removed using Z-Buffer algorithm
  - Patches sorted by distance from radar
  - Any patch facing backwards or directly behind another is removed
- Response at a single range gate is sum over all patches in range gate

ILLUMINATION

Illumination from different looks





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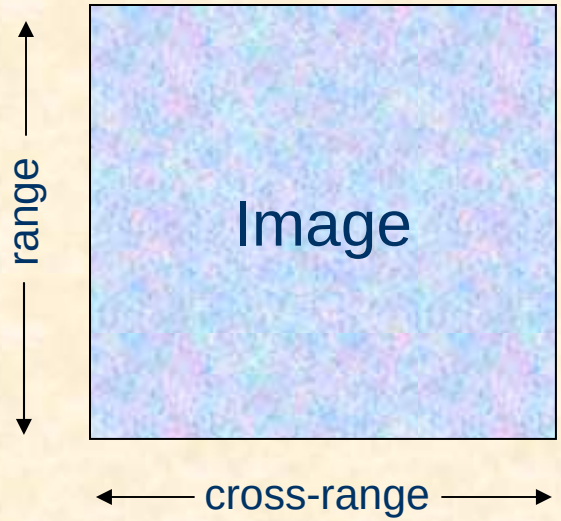
BASIC RADAR IMAGING

Why does SAR imaging work in the first place?

- We assume there exist fast orthogonal transforms that take data into uncorrelated components in radar-centered (range, cross-range) image space.



Range: matched filtering or pulse compression, implemented with FFT

Cross-Range: Doppler processing, or spectrum estimation via FFT

COMPUTATIONAL COMPLEXITY OF SAR

N x N desired image

N range bins, N pulses in CPI

Range processing: $N (N\log_2 N)$ flops

Doppler processing: $N (N\log_2 N)$ flops

Total computational complexity $O(N^2\log_2 N)$
Same as 2-D FFT

OUR COMPUTATIONAL COMPLEXITY

- Terrain relief
- Multiple viewpoints
- Illumination patterns
- Earth-centered coordinates

All mathematical structure leading to the FFT is destroyed

SAR MODEL

- **z**: data vector
- **u**: underlying random vector, indexed by radar coordinates
- Model: $\mathbf{z} = \mathbf{A}\mathbf{u} + \mathbf{n}$
- Inversion: $\mathbf{u} = \mathbf{A}^{-1}\mathbf{z}$ (fast, and only once)
- Resulting image – squared magnitude of **u** components

OUR MODEL

- $\mathbf{z_k}$: data vector (k = viewpoint index)

- $\mathbf{u_k}$: underlying random vectors, indexed by *Earth* coordinates

- Model: $\mathbf{z_k} = \mathbf{A_k}\,\mathbf{L_k}\mathbf{u_k} + \mathbf{n_k}$, $k = 1 \ldots K$

  $\mathbf{u}_k \sim CN(0,\Gamma)$

  - Different A_k
  - A_k not invertible, not square, full
  - No nice mathematical structure

OUR MODEL

EM Algorithm: at each step, compute the conditional mean and covariance of u_k given z_k and the current iterate $\Gamma^{(p)}$.

Rough numbers: for N pixels in each direction, and partitioning of data across ranges, this requires solution of N NxN systems for each iteration and each viewpoint.

N^4 does not compare very well with $N^2 \log_2 N$!!

OUR MODEL

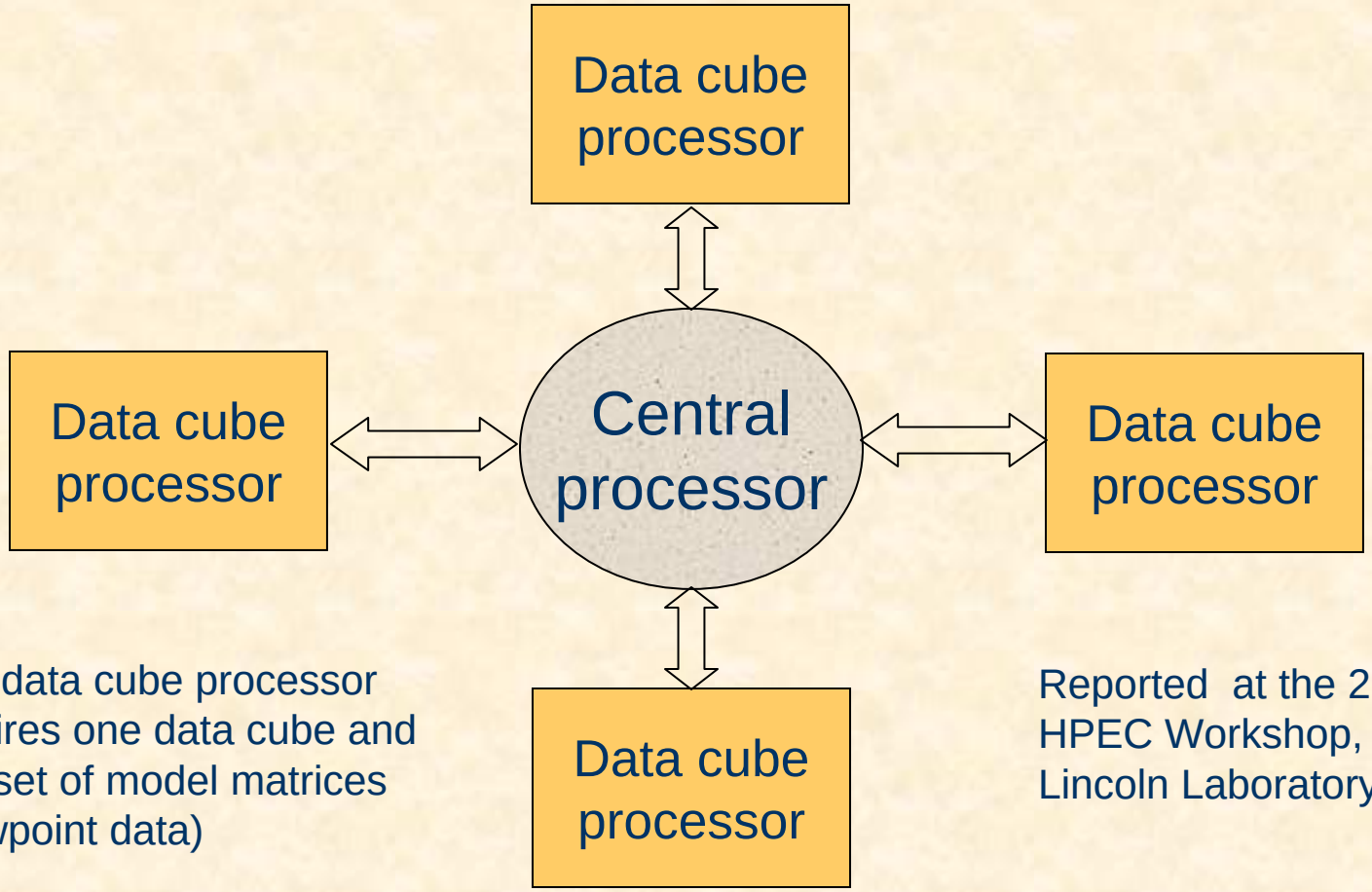
For our full-scale simulation N ~ 500

- For each EM iteration and each viewpoint, we were solving 500 500x500 linear systems for each iteration (on average).

- *No surprise*: even with 2 sec per system, we were using 20 minutes on 2GHz Dell PC running MATLAB, per iteration, per viewpoint.

ONE PIECE OF GOOD NEWS

EM Algorithm parallelizes nicely across data cubes



One data cube processor requires one data cube and one set of model matrices (viewpoint data)

Reported at the 2004 HPEC Workshop, MIT Lincoln Laboratory

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SOLUTION: CROSS-RANGE QUANTIZATION

- Problem in radar coordinates (range, angle, Doppler) has a lot of mathematical structure
- We were already quantizing in range: after range processing (matched filtering) contribution of each pixel assumed to belong to only *one* range bin.
- Extend this idea to angle and Doppler, *i.e.,* cross-range.
- Pre-process data using FFTs (2-D FFT since both angle and Doppler contribute cross-range information)
- Then make key assumption:

Each ground patch contributes to only one range/angle/Doppler bin

ASIDE: SPACE-TIME METHODS FOR RADAR IMAGING?

- In traditional radar processing (1 sensor), # pulses in CPI = # cross-range pixels. Could be Doppler-ambiguous. If PRF is high, data is range-ambiguous.

- With space-time data, suppose there are M_s sensors and M_d pulses in one-CPI data cube. Up to $M = M_s M_d$ cross-range pixels.

- Could lower PRF and eliminate range ambiguities, and at the same time use multiple sensors to resolve Doppler ambiguities. Every cross-range index has a unique $\mathbf{v}_s \times \mathbf{v}_d$ signature vector.

This idea needs a lot more exploration!

WHAT DOES QUANTIZATION BUY US?

- New problem, after coordinate transformation and quantization:

$$\mathbf{z}_k \sim CN\left(\mathbf{0}, \mathbf{A}_k \Gamma \mathbf{A}_k^{\mathrm{H}} + \sigma^2 \mathbf{I}\right)$$

$$\mathbf{u}_k \sim CN(0, \Gamma)$$

- Both $\mathbf{A}_k \Gamma \mathbf{A}^{\mathbf{H}}{}_k$ and Γ are diagonal!

- Define: $\mathbf{r}_k = diag\{\mathbf{R}_{\mathbf{z}}(k))\}$

$$\gamma = diag\{\mathbf{\Gamma}\}$$

$\mathbf{B}_k$ is very sparse – one nonzero element per column

$$\mathbf{r}_k = \mathbf{B}_k \gamma + \sigma^2 1$$

WHAT DOES QUANTIZATION BUY US?

- Log-likelihood, the gradient of the log-likelihood and basic EM step are easily computed for this new model
- *Resulting computational complexity is linear in the number of pixels!*
- Improvement on the order of N^2
- Example: N=500, improvement factor 250,000

CONCLUSIONS

- Presented problem of radar imaging from multiple viewpoints and multiple noncoherent data sets as a maximum-likelihood structured covariance estimation problem
- Derived and implemented EM Algorithm
- Included stable SVD methods, ordered-subset (OS) algorithm, parallel implementation, land-use aggregration
- Proof-of-concept through medium-scale simulation
- Computational complexity of full-scale simulation unmanageable
- Solution: quantization of every ground patch in range, angle, and Doppler. Orders of magnitude reduction in computational complexity.
- Space-time methods for radar imaging is a topic that needs to be explored independently.