Degrees of Freedom Inherent in Knowledge Aided RF Urban Mapping

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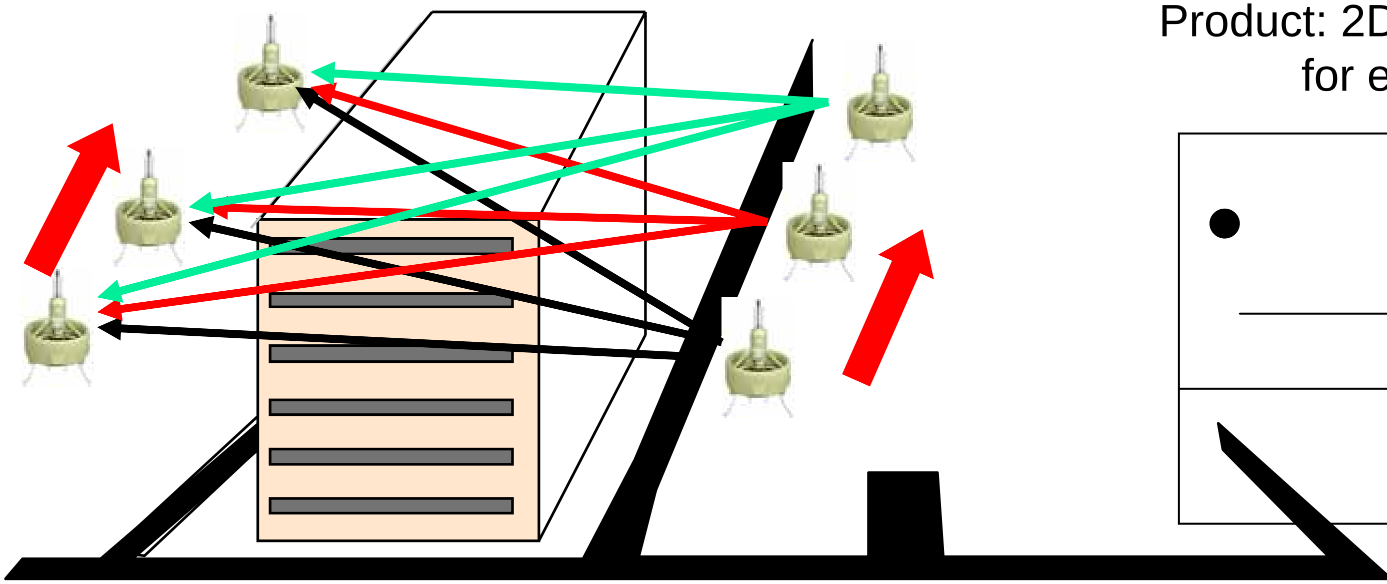
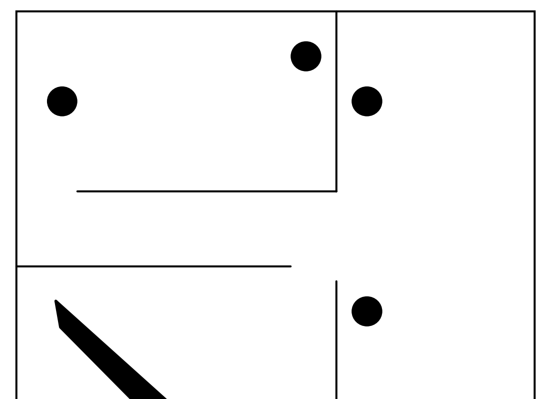
Urban RF Imaging-Degrees of Freedom

- **Problem addressed:**
  - **How does angular diversity and frequency impact mapping capability (resolution) ?**
    - Bistatic measurements
      - How many transmit/receive pairs required?
      - Ho much bandwidth required?
- Approach: Eigen analysis of the inversion operation for the inverse source problem



Product: 2D scattering map for each floor

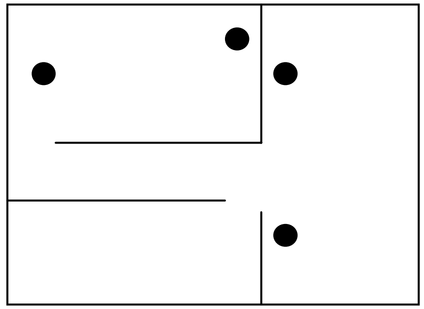


Study Goals

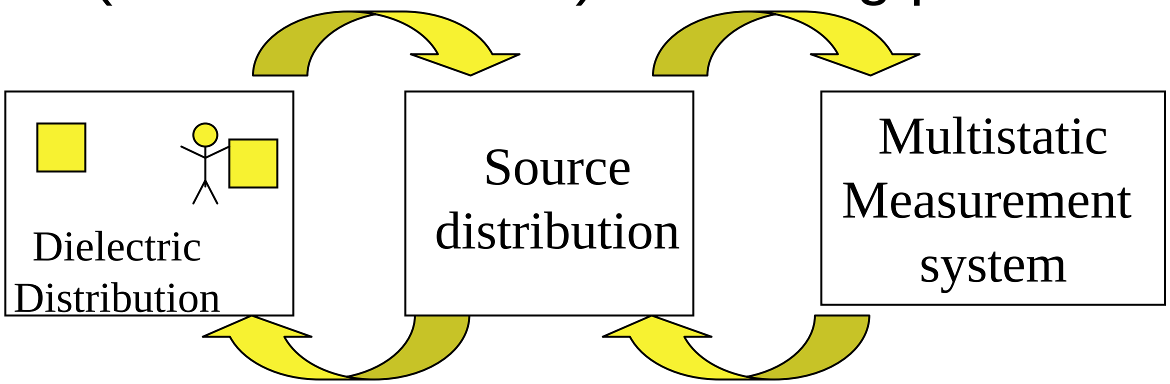
- Goals:
  - Analyze the various degrees of freedom to the urban estimation and detection problem.
    - number, type and contribution
  - Consider the role of prior information in helping to reduce the number of collected and processed degrees of freedom.
    - maps, floor plans, prior imagery, parametric knowledge of phenomenology
- Approach:
  - Information theoretic framework
    - eigen analysis
    - Cramer-Rao estimation bounds
    - Chernoff detection/false alarm bounds
- Scenario:
  - Simple moving human scenario in an enclosed, walled environment

Overview of this Inverse Problem

- Pictorial

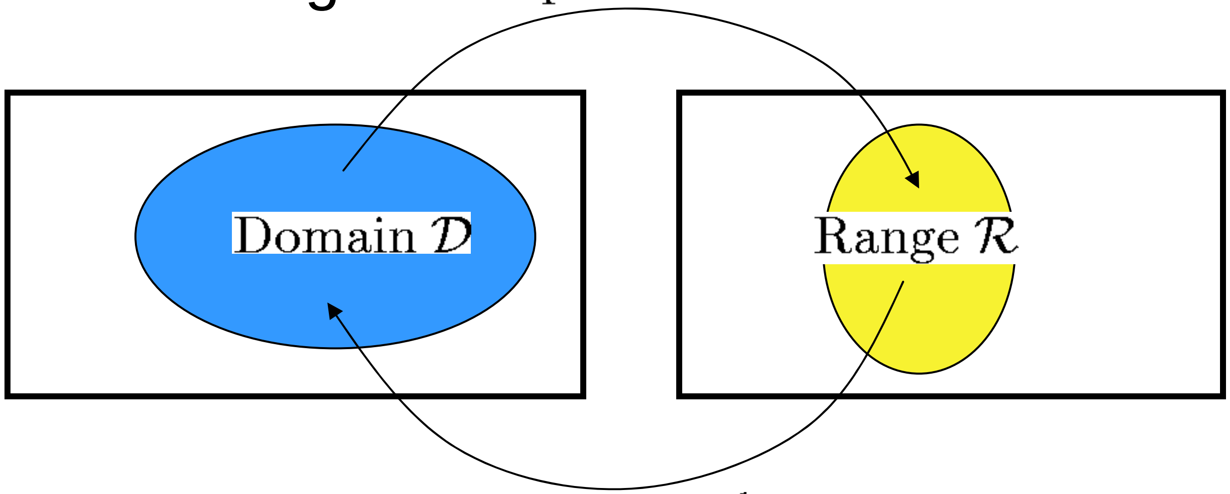


- Collect multistatic measurements of building with objects
  - Metal, concrete, wood, humans, other
- Inverse (discrimination of) scattering problem



Inverse Problems

- Operator Diagram



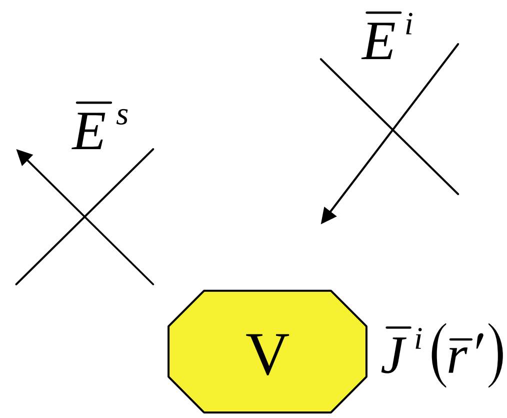
- Mathematical setup : $K : \mathcal{D} \to \mathcal{R}$
  - Important properties of forward operator
    - One-to-one (possibility of inverse)
    - Onto (corresponds to determining the range)
    - Compact or not (possibility of bounded inverse)

Basic EM Equations

Direct Source/Inverse Source Problem

$$\overline{E}^{r}(\overline{r}) = \int_V \overline{\overline{G}}(\overline{r}-\overline{r}')\overline{J}(\overline{r}')dr'$$

observation source



Scalar $\overline{\overline{G}}(\overline{r}-\overline{r}') = \begin{cases} -(j/4)H_o^{(2)}(k|\overline{r}-\overline{r}'|) & \text{2D} \\ \exp(jk|\overline{r}-\overline{r}'|)/(4\pi|\overline{r}-\overline{r}'|) & \text{3D} \end{cases}$

Direct Scattering Problem

$$\overline{E}(\overline{r}) = \overline{E}^{i}(\overline{r}) + \int_V \overline{\overline{G}}(\overline{r}-\overline{r}')\overline{J}^{i}(\overline{r}')dr' \qquad \overline{E} = \overline{E}^{i} + \overline{E}^{s}$$

$$\overline{E}^{s}(\overline{r}) = \int_V \overline{\overline{G}}(\overline{r}-\overline{r}')\overline{J}^{i}(\overline{r}')dr' \qquad \overline{r} \notin V$$

$$\overline{E}^{i}(\overline{r}) = \overline{E}(\overline{r}) - \int_V \overline{\overline{G}}(\overline{r}-\overline{r}')\overline{J}^{i}(\overline{r}')dr' \qquad \overline{r} \in V$$

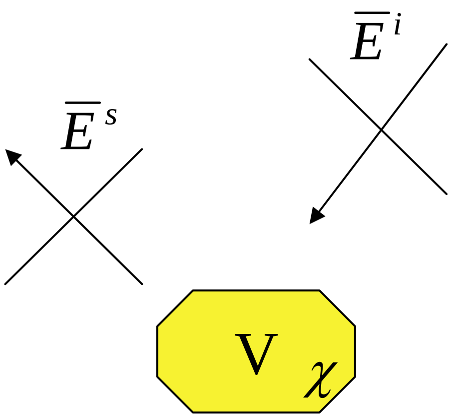
These are linear problems

Inverse Scattering Problem

Define complex relative permettivity

to background

$$\chi(\bar{r}) = \varepsilon_{rb}(\bar{r}) - 1$$



$$\bar{E}^{s}(\bar{r}) = j\omega\varepsilon_{b}\int_{V}\bar{\bar{G}}(\bar{r}-\bar{r}')\chi(\bar{r}')\bar{E}(\bar{r}')dr' \qquad \bar{r} \notin V$$

$$\bar{E}^{i}(\bar{r}) = \bar{E}(\bar{r}) - j\omega\varepsilon_{b}\int_{V}\bar{\bar{G}}(\bar{r}-\bar{r}')\chi(\bar{r}')\bar{E}(\bar{r}')dr' \qquad \bar{r} \in V$$

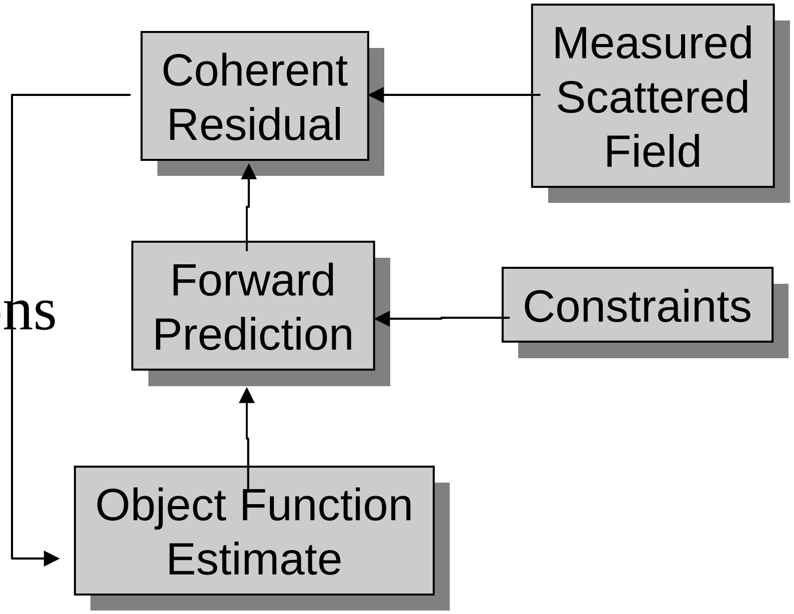
$$\bar{E}^{s} = A_{e}(\chi\bar{E}) \qquad\qquad \bar{E} = \bar{E}^{i} + \bar{E}^{s}$$

$$\bar{E}^{i} = \bar{E} - A_{i}(\chi\bar{E}) = (I - A_{i}\chi)\bar{E}$$

Inverse scattering problem is non-linear

Iterative non-linear optimization approach

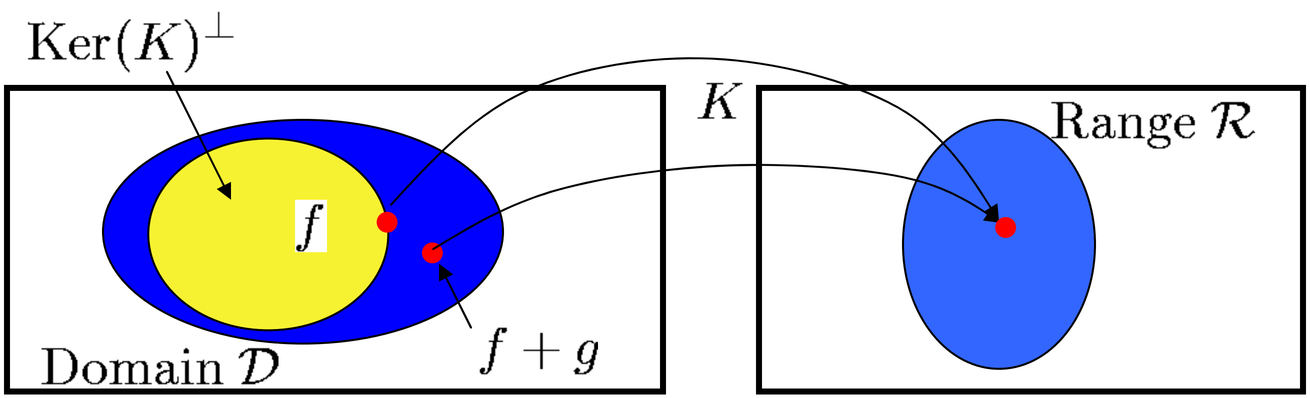
- Sequence of steps with increased spatial resolution and accuracy
  - Initial steps invoke a linear model over large synoptic view
  - Non-linear model invoked over local areas
- Perform the inverse source problem first to find ‘bright objects’ – a reflectance map
  - Indicative of metal
  - Noninteractive model
- Estimate location of potential objects
  - Invoke model with field interactions
  - Knowledge of building materials and shapes
  - Constraints on operator domain
    - Sparse objects
    - Context
- Perform hypothesis test for ‘human like’ objects



Linear Inverse Problems - Background

- Suppose have $K : \mathcal{D} \subset \mathcal{H} \rightarrow \mathcal{R} \subset \mathcal{K}$ where
  - Where $\mathcal{H}, \mathcal{K}$ are Hilbert (inner product) spaces
    - $\mathcal{D}$ is a closed linear subspace (implies range is linear)
  - Operator K is bounded and linear
  - Important subspaces in domain are the kernel of *K* and its perpendicular complement

$$\begin{aligned} \mathrm{Ker}(K) &= \{g \in \mathcal{H} : Kg = 0\} \\ \mathrm{Ker}(K)^{\perp} &= \{f \in \mathcal{H} : \langle f, g \rangle = 0 \quad \text{for all } g \in \mathrm{Ker}(K)\} \end{aligned}$$



Linear Inverse Problems$_2$

- For linear inverse problem, we are blind to components in $\mathrm{Ker}(K)$
  - Only mitigation is a priori information (non-negativity, smoothness, other constraints)
  - More relevant background for inverse problems
- Typical (important) properties of inverse problems
  - Dimension of domain is infinite and dimension of range is finite (automatically implies that cannot be one-to-one)
  - Operator K, restricted to $\mathrm{Ker}(K)^\perp$ is compact, i.e.,

$$\inf_{f \in \mathrm{Ker}(K)^\perp} \frac{\|Kf\|}{\|f\|} = 0$$

    - Implication is that for any vector/function f can find a vector/function g where $\|g\| = 1$ and $K(f+g)$ is arbitrarily close to $K(f)$
    - Even with one-to-one, have problems – unbounded inverse

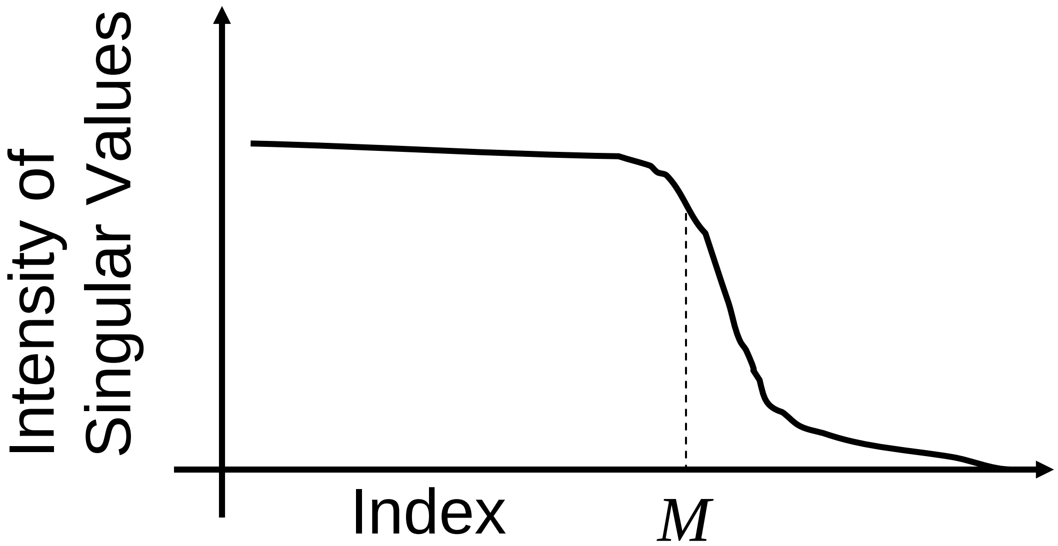
Linear Inverse Problems$_3$

- Concentrate on case where K is one-to-one
- Linear inverse problem with compact operators are ill-posed (unbounded inverse)
  - Even with the unique inverse, noise is amplified because of the unbounded inverse
  - Very active area of ongoing research/applications
- Very useful framework for analyzing ill-posed problems and developing "stable" inverse algorithm is the SVD

$$
\begin{aligned}
Kf &= \sum_{j=1}^{\infty} \mu_j v_j \langle f, f_j \rangle \\
\{f_j\} &= \text{singular vectors - orthonormal basis in } \mathcal{D} \\
\{v_j\} &= \text{singular vectors - orthonormal basis in } \mathcal{R} \\
\{\mu_j\} &= \text{singular values} - \text{positive and non-increasing}
\end{aligned}
$$

Degrees of Freedom of Scattered Field

- Analytic properties of radiation operator imply kernel imply singular values have step-like behavior, with exponential decay after knee
- Thus scattered field can be accurately approximated by a finite number M of singular functions – the number is essentially the degrees of freedom of the field [Bucci and Franceschetti, 1989]



Approximate Formula for Degrees of Freedom (DoF) of Radiated Field-Single Freq.

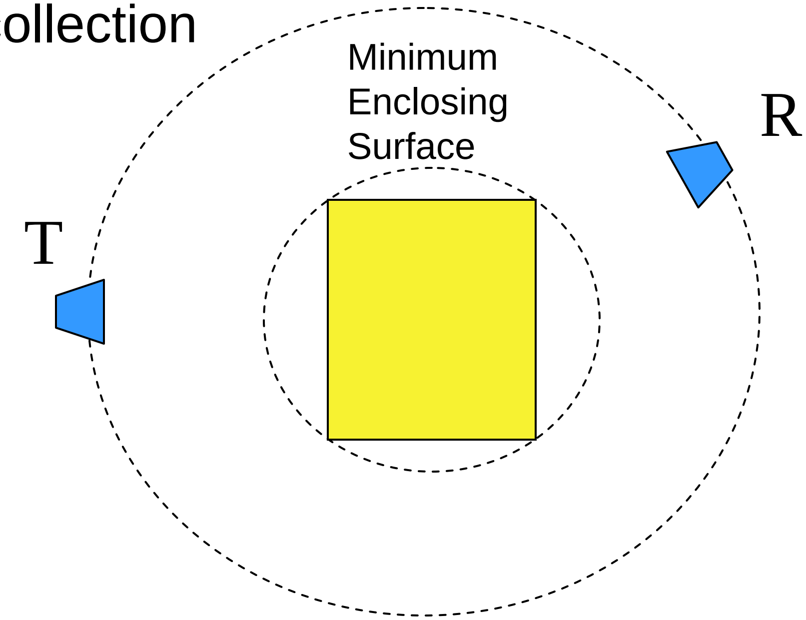
- Equivalent estimates of DoF can be computed using SVD of radiation operator or Effective Bandwidth approach
- For a source enclosed by a sphere (circle for 2-D), Bucci and D'Elia [1996] give maximum **radiated field** DoF as

$$M \sim (\text{Minimum Sphere Area}) / (\lambda / 2)^2 \quad \text{for 3-D}$$

$$M \sim (\text{Minimum Circle Circumference}) / (\lambda / 2) \text{ for 2-D}$$

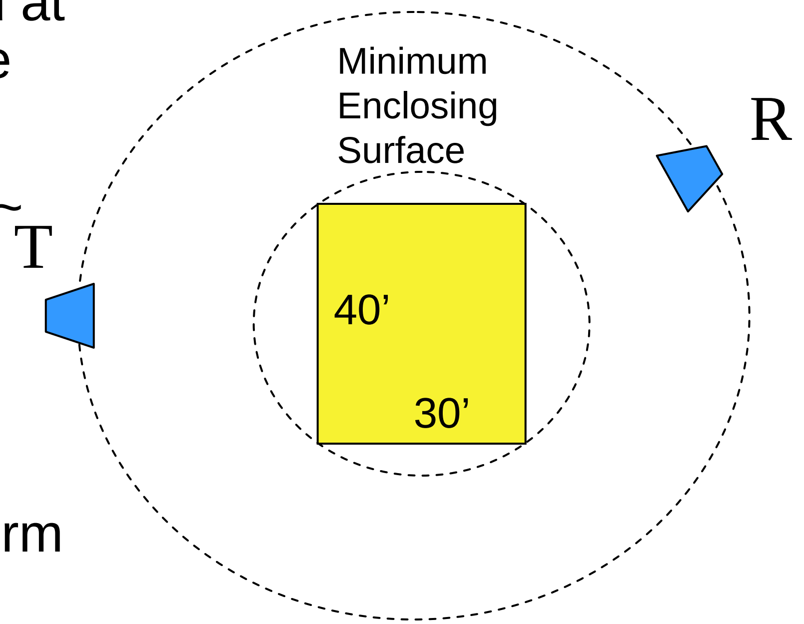
- DoF for **measurement** depend on collection
  - M for single illumination
  - $M^2/2$ for full bistatic

DoF bound amount of target information (number of parameters) that can be estimated from measurement



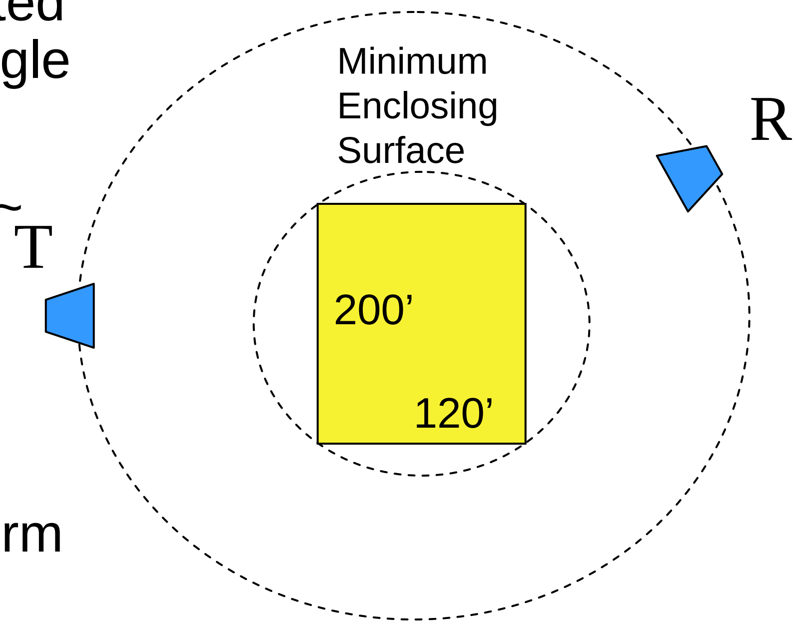
House Example

- Assume 30’ x 40’ house interrogated at 1 GHz by sensors rotating in a single plane about target (~ 2-D geometry)
- Radiated field single bistatic DoF $M \sim$ 319
  – Implies 319 parameters could be estimated for single illumination
  – 50881 parameters for full bistatic
- Assume we divide building into uniform square regions where we estimate a dielectric constant and conductivity, e.g. 2 parameters per region
- DoF imply we could spatially sample the building at
  – ~ 33” for single bistatic illumination
  – ~ 23” for monostatic
  – ~ 2.5” for full bistatic illumination



Building Example

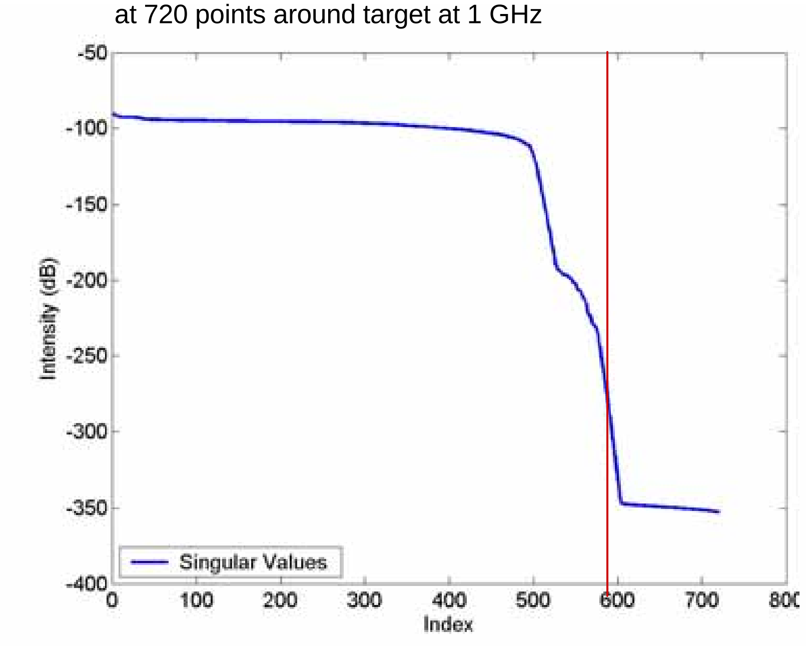
- Assume 200’ x 120’ house interrogated at 1 GHz by sensors rotating in a single plane about target (~ 2-D geometry)
- Radiated field single bistatic DoF $M \sim$ 1490
  - Implies 1490 parameters could be estimated for single illumination
  - 1110050 parameters for full bistatic
- Assume we divide building into uniform square regions where we estimate a dielectric constant and conductivity, e.g. 2 parameters per region
- DoF imply we could spatially sample the building at
  - ~ 64” for single bistatic illumination
  - ~ 48” for monostatic
  - ~ 2.5” for full bistatic illumination



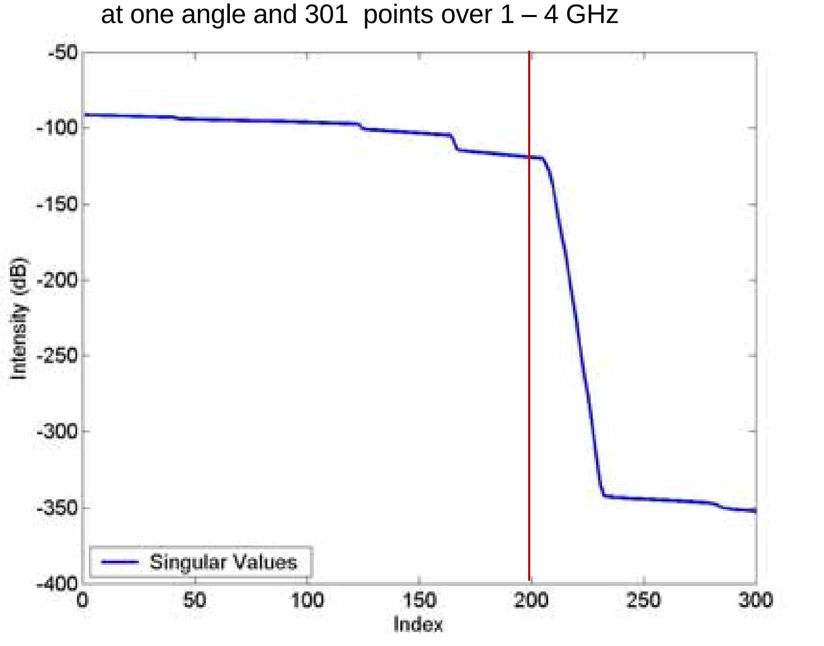
Number of Degrees of Freedom Can Be Increased Using Frequency

- Number of significant singular values approximately equal to unambiguous angle and frequency sampling of target scattering

Singular values for operator describing backscattering from 10 x10 m target measured at 720 points around target at 1 GHz



Singular values for operator describing backscattering from 10 x10 m target measured at one angle and 301 points over 1 – 4 GHz



4ka=angle measurement range/required unambiguous angle sampling for target ~ 592

2BD/c ~frequency measurement range/required unambiguous frequency sampling for target along LOS ~ 200

- Preliminary analysis shows that increase is: L(object)/res.

Applications of SVD to Inverse Problems

- Implications of SVD

$$Kf = \sum_{j=1}^{\infty} \mu_j v_j \langle f, f_j \rangle$$

  – The inverse operator exists and is given by

$$K^{-1}v = \sum_{j=1}^{\infty} \frac{1}{\mu_j} \langle v, v_j \rangle f_j$$

  - With noisy/quantized measurement of $Kf + N$, blindly inverting generates

$$K^{-1}(Kf + N) = f + \sum_{j=1}^{\infty} \frac{\langle N_j, v_j \rangle}{\mu_j}$$

    – Variance of noise terms going to infinity
  - Regularization refers to a class of techniques trying to address the “noise amplification” problem above

Applications of SVD to Inverse Problems

- Implications of SVD

$$Kf = \sum_{j=1}^{\infty} \mu_j v_j \langle f, f_j \rangle$$

  - Compactness implies that
    - Singular values converge to 0
    - Range of the operator is a linear subspace, but it is not closed
      - Picard condition: a vector v is in the range of operator if and only if

$$\sum_{j=1}^{\infty} \frac{|\langle v, v_j \rangle|^2}{\mu_j^2} < \infty$$

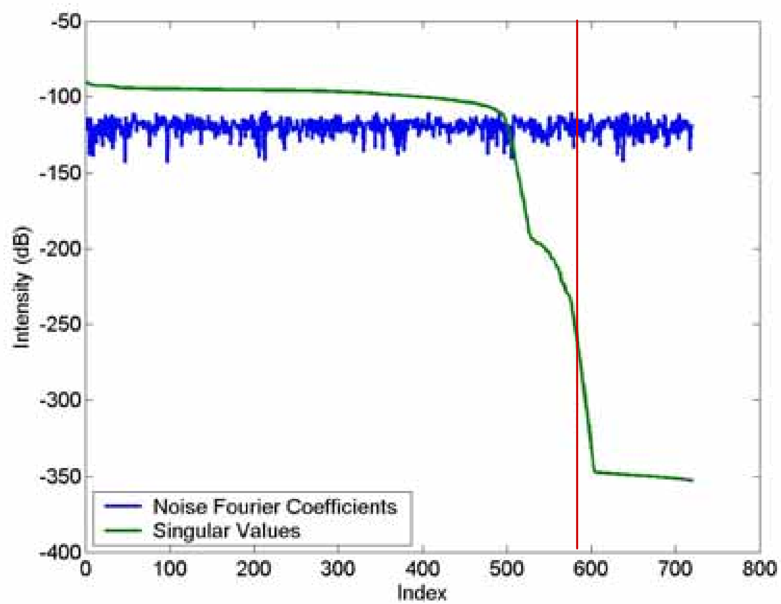
      - Note that in general, a vector v satisfies that

$$\sum_{j=1}^{\infty} |\langle v, v_j \rangle|^2 < \infty$$

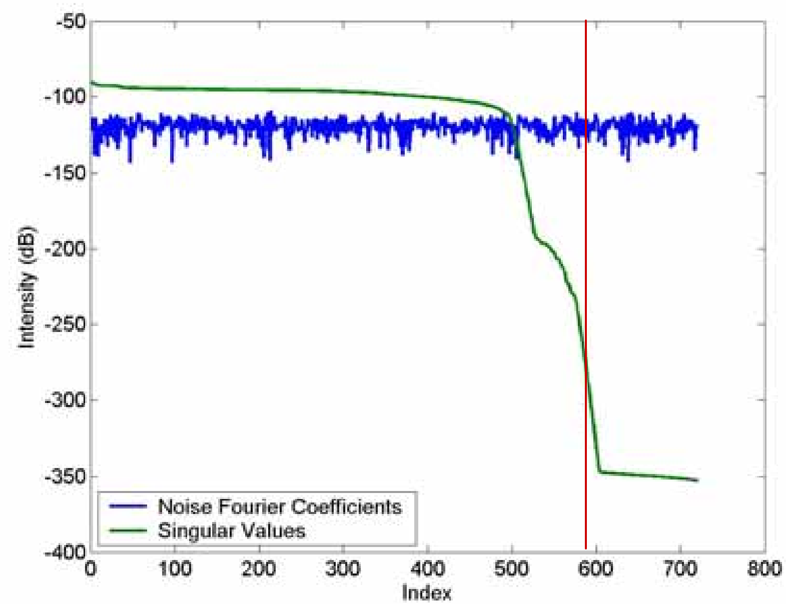
      - Often said that "Fourier coefficients" of vectors in range converge to 0 at a rate of faster than $\mu_j j^{-\frac{1}{2}}$

Comparison of Singular Values to Fourier Coefficients

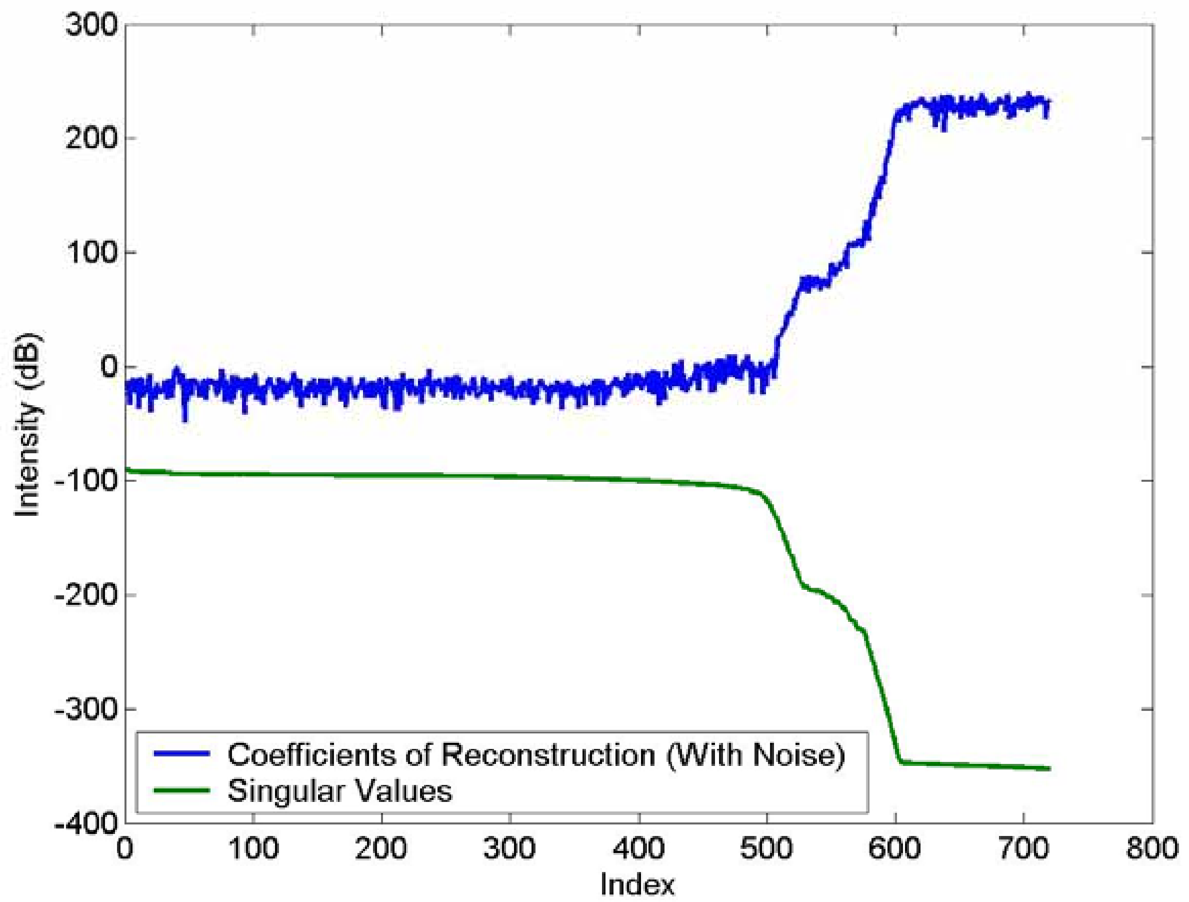
- SVD of operator that maps 1681 sources distributed over 10 x 10 m rectangle to 720 backscattering measurements on 100 m circle about target
- Single scattering operator normalized such that one source produces the same received power as a single discrete scatterer with a cross-section of equivalent value
- Example has a single measurement SNR of 6 dB



4ka ~ 592



Noise Amplification



Regularization for Inverse Problems

- With noisy data

$$d = Kf + N$$

  - Truncated SVD

$$R_\alpha d = \sum_{\{|\langle d, v_j\rangle| \geq \alpha\}} \frac{1}{\mu_j} \langle d, v_j\rangle f_j$$

  - Tichonov

$$R_\alpha d = \sum_{j=1}^{\infty} \frac{\mu_j}{\mu_j^2 + \alpha} \langle d, v_j\rangle f_j$$

- Regularization parameters chosen based on level of noise and expected smoothness/falloff of Fourier coefficients

Uniqueness of Inverse Discrimination Problem

- Main objective is the discrimination between classes of objects (humans vs. other classes)
  - Depending on the classes and a priori information, discrimination could be significantly better posed than the full inverse problem
  - Have classes of objects

$$\begin{aligned} \mathcal{F}_o &= \{\text{buildings/rooms with no human}\} \\ \mathcal{F}_1 &= \{\text{buildings/rooms with humans}\} \end{aligned}$$

    - Want discrimination between classes
      - First order ability to discriminate (for the white noise case, this is the statistically right measure) is

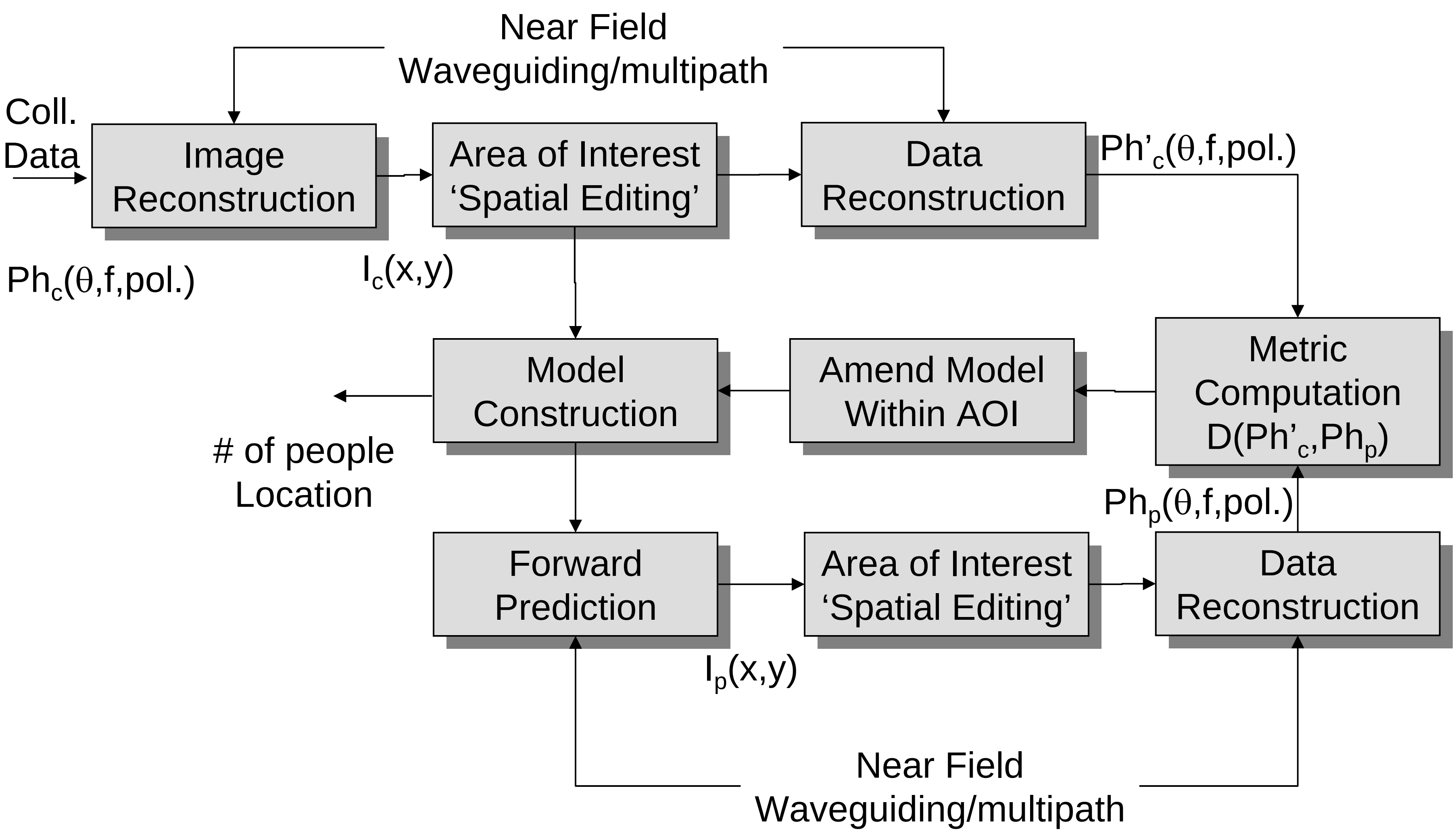
$$D_K(\mathcal{F}_o, \mathcal{F}_1) = \inf\{\|Kf_o - Kf_1\| : \; f_o \in \mathcal{F}_o, f_1 \in \mathcal{F}_1\}$$

      - Not taking into account complexity/computability of discrimination algorithm
    - Dynamic range is issue -- Kf_o vs. $Kf_o - Kf_1$

Exploitation of Sparse Representations

- Logan/Donoho/Stark (others) have identified and quantified the utility of signal/image sparseness
  - When signal and/or noise representations span a space with a significantly smaller dimension than the number of measurements/DOF, much more accurate estimation (discrimination) is possible (form of *a priori* information)
  - In papers, referred to as super-resolution but this is a bit of a misnomer
  - In many important scenarios, a significant portion of our image volume is not occupied
    - Sparse representation which can be exploited
  - Will exploit sparsity of representation where appropriate
  - Reference
    - Superesolution via sparsity constraints (Donoho, SIAM 1992)
    - Signal recovery and the large sieve (Donoho and Logan, SIAM 1992)
    - Uncertainty principles and signal recovery (Donoho and Stark, 1989)

GLRT Approach



Parameters and Hypotheses

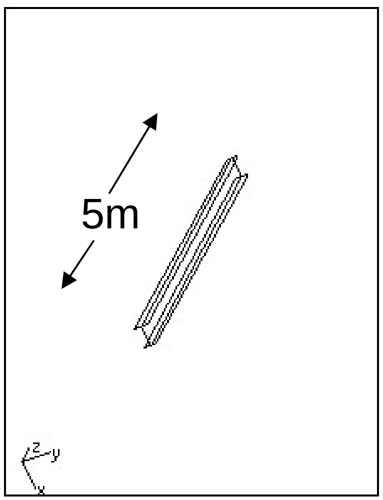
Highly restricted discrete hypothesis set of canonical shapes and materials

Parameter	Hypothesis Space	Observables
Complex Permitivity, ε	Metal, Saline, Muscle, Wood, Board, Plastic, Metal	Frequency, Angle
Shape	Ellipsoid, Cylinder, Box, Beam	Angle, Polarization
Position	Location within res. Cell	Reflectivity, phase

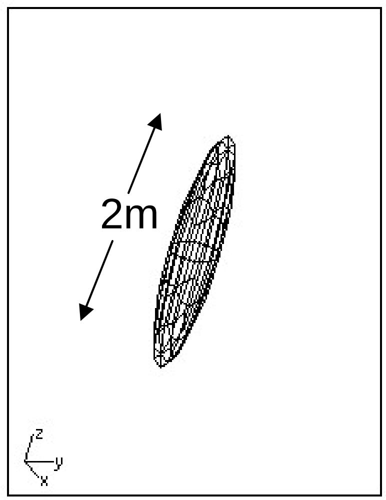
Comparison of Bistatic Scattering

- The bistatic scattering from a 5 m long metal I-beam and a 2 m long dielectric ellipsoid (e_r = 50 – j 30) were computed at 1 GHz using WIPL-D MOM code
  - Incident field broadside to targets and polarized parallel to target's long axis (z-axis)

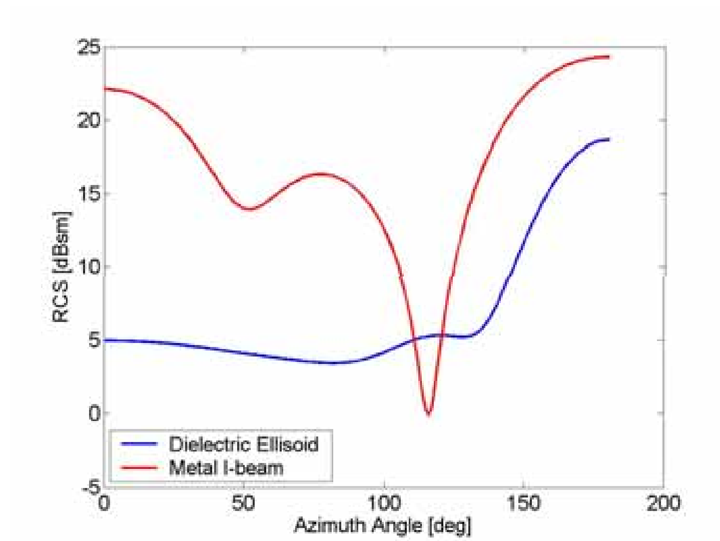
I - beam



Ellipsoid

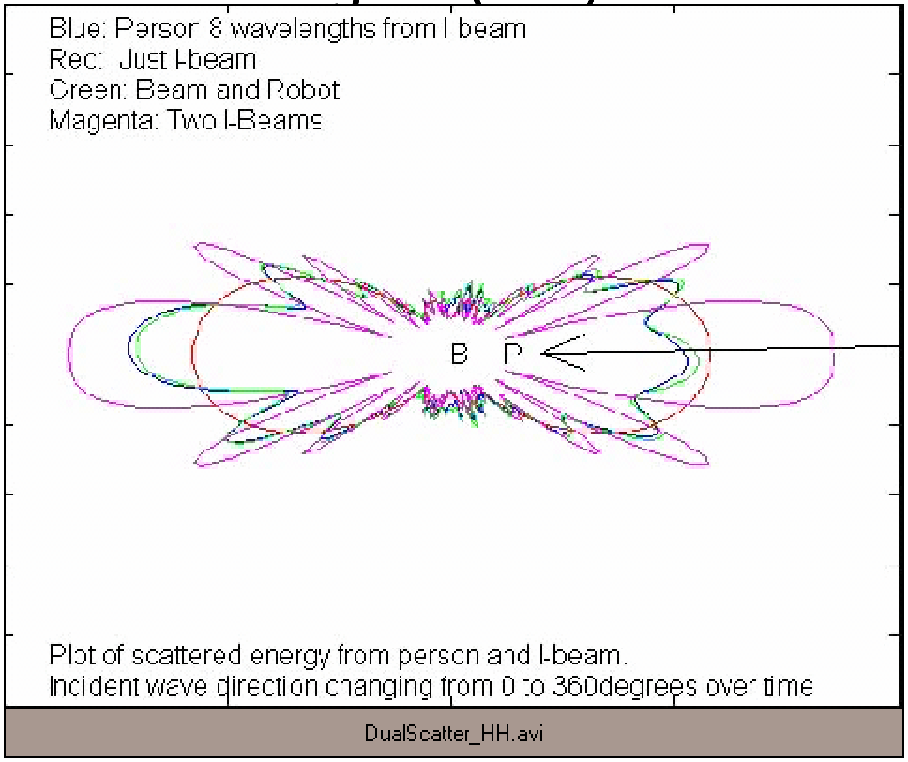


Bistatic Scattering



Movie of Reflected Energy

- Using every combination of transmitter and receiver locations on circle around target
- Shown with person 8 wavelengths (blue) and 7 wavelengths (red) from I-beam. 1GHz excitation



- Explore the contribution of material and shape
- Single beam, two beams, ‘metal person’, dielectric person
- Shows that shape is the major driver in reflectivity maps

Human Detection: Two Step Algorithm

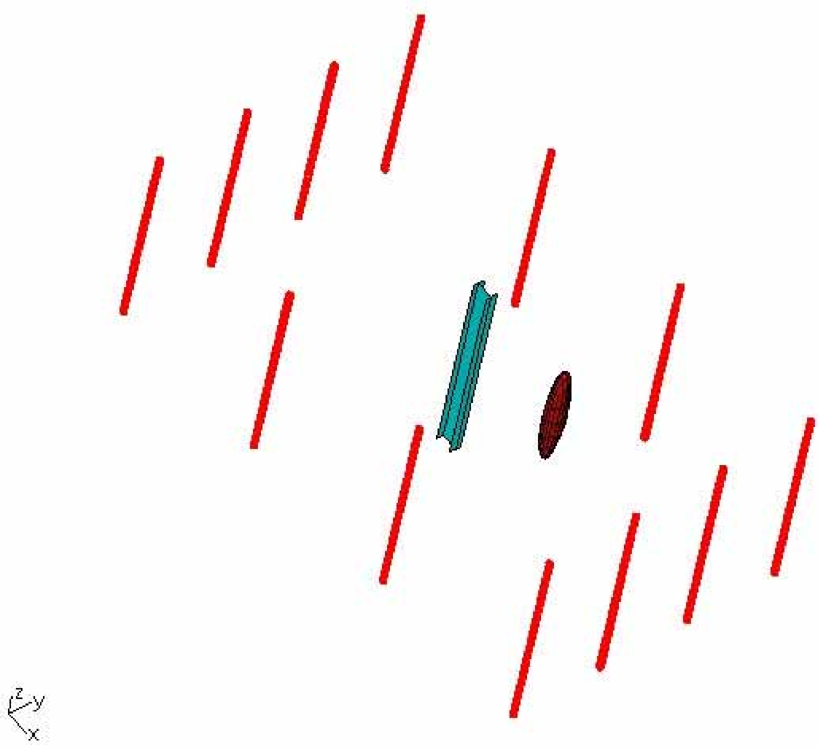
- **Step 1:** Determine if the approximate shape of an object is more similar to a person or to clutter, such as an I-Beam
  - Shape has largest effect on data, more so than dielectric

Once the shape has been determined to be similar to a person

- **Step 2:** Test dielectric constant
  - If the dielectric is low, the object is deemed likely to be human

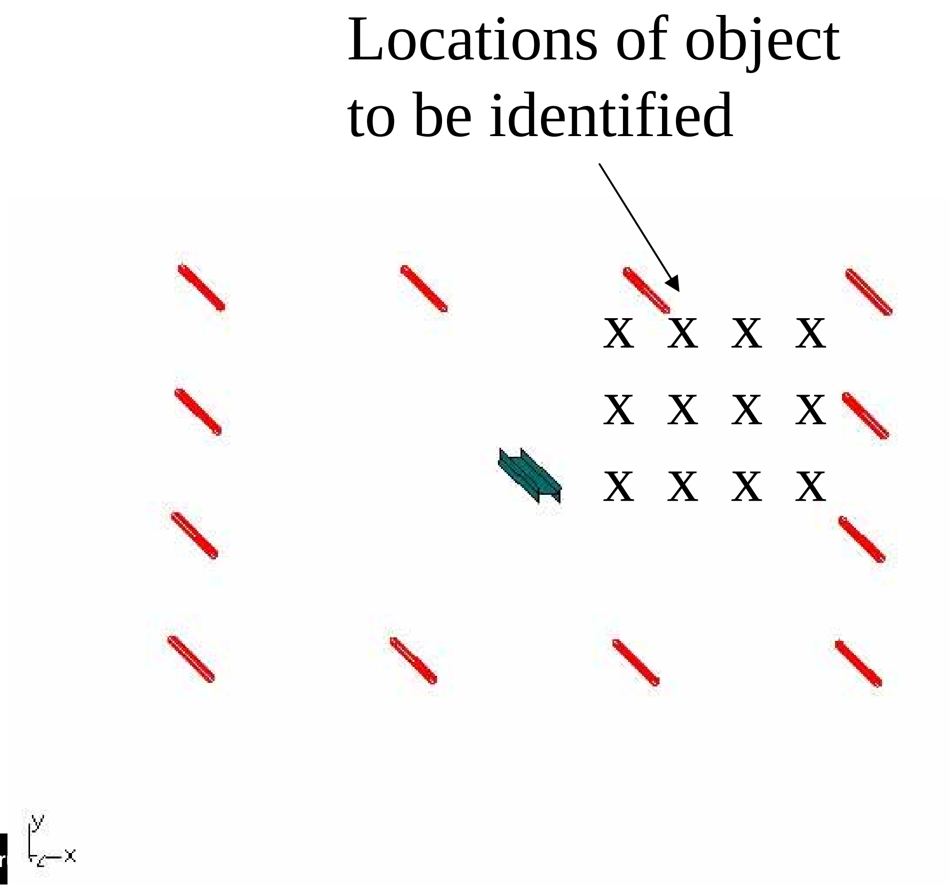
Simulation of Room With Wire Walls

- Simulation model was of
  - 3.5m tall, perfect conductor (PEC) beam in room center
  - Object to be Identified is 2m tall ellipsoid “person” (50-30i ε dielectric)
  - 12, 3.5m tall PEC wires as walls
- Simulation run a 457 angles, ranging 360 degrees fully bi-static, using WIPL-D, at 1GHz.
- Scene extent is 10.9m
- Room is 9m by 5m by 3.5m tall



Positions of Object to be Identified

- The object to be identified (person) is located in each of 12 positions
- Positions are offset from center beam in X direction by 1 to 4 meters (in 1 meter increments)
- Positions are offset from center beam in Y direction by 0 to 2 meters (in 1 meter increments)

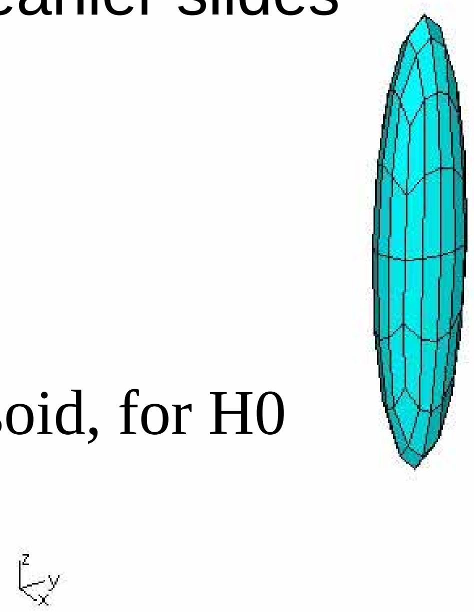


Stage 1 Detection: Spatial Editing Provides Gain

Matched Filter H0 (Using CROPPED data)	Matched Filter H1 (Using CROPPED data)	Disp. X	Disp. Y	Scene (Using CROPPED data)	SNR
Beam in free space	Person in free space	0	0	Person in free space	68.03 dB
Beam in free space	Person in free space	1	0	Person, in wire room	67.6 dB
Beam in free space	Person in free space	1	1	Person, in wire room	67.8 dB
Beam in free space	Person in free space	1	2	Person, in wire room	66.1 dB
Beam in free space	Person in free space	2	0	Person, in wire room	67.9 dB
Beam in free space	Person in free space	2	1	Person, in wire room	67.9 dB
Beam in free space	Person in free space	2	2	Person, in wire room	65.9 dB
Beam in free space	Person in free space	3	0	Person, in wire room	67.9 dB
Beam in free space	Person in free space	3	1	Person, in wire room	67.9 dB
Beam in free space	Person in free space	3	2	Person, in wire room	67.9 dB
Beam in free space	Person in free space	4	0	Person, in wire room	67.2 dB
Beam in free space	Person in free space	4	1	Person, in wire room	65.8 dB
Beam in free space	Person in free space	4	2	Person, in wire room	66.0 dB
9.5 dB Gain using Spatial Editing					Average of SNRs: 67.2 dB

Stage 2: Test for Low Dielectric Const. Object

- H1 is “Person” in free space
- H0 is “Robot” in free space
  - Robot is same as person, except perfect conductor
- This test is run after the shape has been determined and known to be more similar to a person than other clutter, such as a beam
- Same cropping, positioning used as in earlier slides



PEC Ellipsoid, for H0

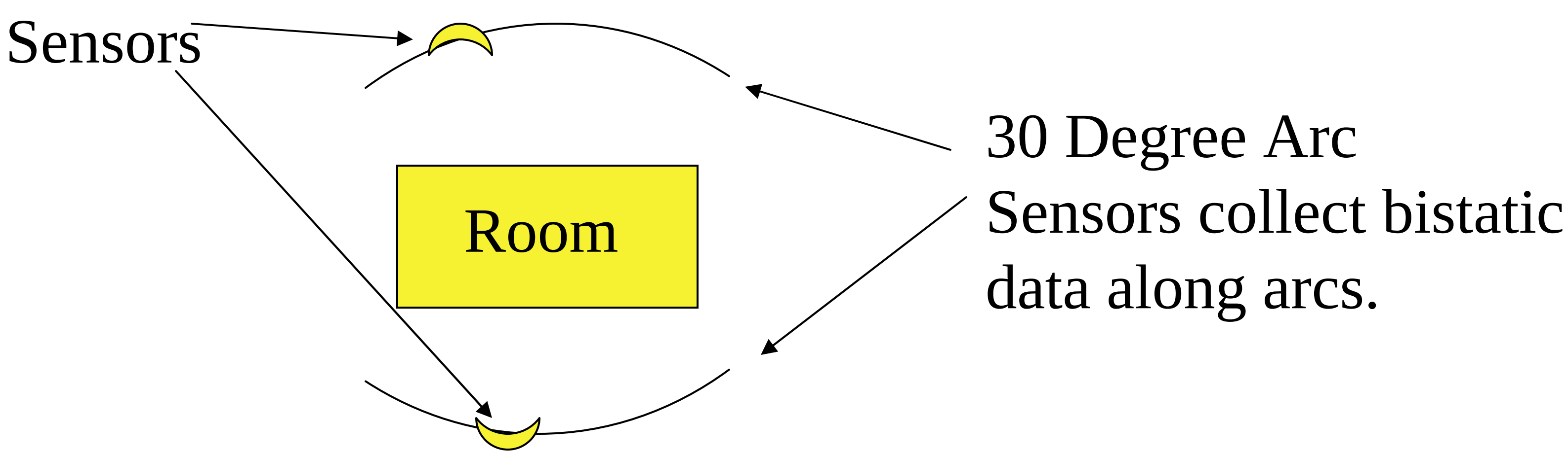
Stage 2: Dielectric Hypotheses Spatial Editing Provides Gain

Matched Filter H0 (Using CROPPED data)	Matched Filter H1 (Using CROPPED data)	Disp. X	Disp. Y	Scene (Using CROPPED data)	SNR
Robot in free space	Person in free space	0	0	Person in free space	45.2 dB
Robot in free space	Person in free space	1	0	Person, in wire room	36.5 dB
Robot in free space	Person in free space	1	1	Person, in wire room	40.0 dB
Robot in free space	Person in free space	1	2	Person, in wire room	31.0 dB
Robot in free space	Person in free space	2	0	Person, in wire room	40.3 dB
Robot in free space	Person in free space	2	1	Person, in wire room	40.9 dB
Robot in free space	Person in free space	2	2	Person, in wire room	29.1 dB
Robot in free space	Person in free space	3	0	Person, in wire room	40.0 dB
Robot in free space	Person in free space	3	1	Person, in wire room	40.0 dB
Robot in free space	Person in free space	3	2	Person, in wire room	40.0 dB
Robot in free space	Person in free space	4	0	Person, in wire room	34.3 dB
Robot in free space	Person in free space	4	1	Person, in wire room	30.2 dB
Robot in free space	Person in free space	4	2	Person, in wire room	30.0 dB
18.5 dB Gain using Spatial Editing					Average of SNRs: 36.0 dB

Reduced Data Collection Scenario

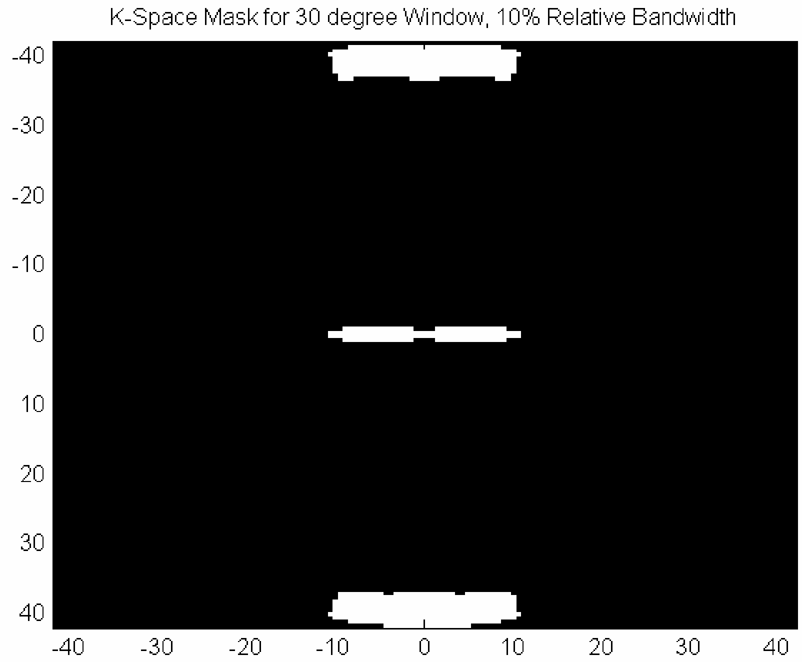
- Same scene as previous slides
- Collection is assumed to occur in a 30 degree arc, both in front and behind the room
- Frequency diversity (Bandwidth) is used to compensate for some of the lost angular diversity
- Simulates building where sides are not accessible

Top Down View Of Scene



Reduced Data Set Acquisition Method

- To save simulation time, the reduced collection scenario was simulated by applying a mask to the full phase history data.
- Only data from the original full bistatic collection which fell within 30 degree angle, and 10% relative bandwidth (fc = 952MHz) was used
- All other data set to zero



Mask applied to Phase Data

Stage 1: Shape Detection (Reduced Collection)Spatial Editing Provides Gain

Matched Filter H0 (Using CROPPED data)	Matched Filter H1 (Using CROPPED data)	Disp. X	Disp. Y	Scene (Using CROPPED data)	SNR
Beam in free space	Person in free space	0	0	Person in free space	56.326 dB
Beam in free space	Person in free space	1	0	Person, in wire room	55.8 dB
Beam in free space	Person in free space	1	1	Person, in wire room	55.9 dB
Beam in free space	Person in free space	1	2	Person, in wire room	55.6 dB
Beam in free space	Person in free space	2	0	Person, in wire room	56.1 dB
Beam in free space	Person in free space	2	1	Person, in wire room	56.1 dB
Beam in free space	Person in free space	2	2	Person, in wire room	55.5 dB
Beam in free space	Person in free space	3	0	Person, in wire room	56.2 dB
Beam in free space	Person in free space	3	1	Person, in wire room	56.2 dB
Beam in free space	Person in free space	3	2	Person, in wire room	56.3 dB
Beam in free space	Person in free space	4	0	Person, in wire room	55.0 dB
Beam in free space	Person in free space	4	1	Person, in wire room	54.8 dB
Beam in free space	Person in free space	4	2	Person, in wire room	55.3 dB
5.9 dB Gain using Spatial Editing					Average of SNRs: 55.7 dB

Restricted data collection reduces ave. SNR by 11.4 dB

Stage 2: Dielectric Hypotheses Image Editing is Crucial for Operation

Matched Filter H0 (Using CROPPED data)	Matched Filter H1 (Using CROPPED data)	Disp. X	Disp. Y	Scene (Using CROPPED data)	SNR
Robot in free space	Person in free space	0	0	Person in free space	28.54 dB
Robot in free space	Person in free space	1	0	Person, in wire room	10.7 dB
Robot in free space	Person in free space	1	1	Person, in wire room	16.7 dB
Robot in free space	Person in free space	1	2	Person, in wire room	17.1 dB
Robot in free space	Person in free space	2	0	Person, in wire room	11.4 dB
Robot in free space	Person in free space	2	1	Person, in wire room	19.5 dB
Robot in free space	Person in free space	2	2	Person, in wire room	13.1 dB
Robot in free space	Person in free space	3	0	Person, in wire room	21.2 dB
Robot in free space	Person in free space	3	1	Person, in wire room	24.3 dB
Robot in free space	Person in free space	3	2	Person, in wire room	19.5 dB
Robot in free space	Person in free space	4	0	Person, in wire room	15.7 dB
Robot in free space	Person in free space	4	1	Person, in wire room	16.0 dB
Robot in free space	Person in free space	4	2	Person, in wire room	14.6 dB
15.3 dB Gain using Spatial Editing				Average of SNRs: 16.7 dB	

Restricted data collection reduces ave. SNR by 19.4 dB

Program Plan

1. **Formulation of the information theoretic bounds.** The pertinent equations will be derived and coded into Matlab.
2. **Seperable DoF analysis:** The free running variables under consideration will be spatial, ***temporal***, ***Doppler***, frequency (center and waveform), and polarization. The scenario of interest will be based on a moving human in an enclosed single story building example.
3. **DoF trade analysis:** DoF subsets will be considered to determine useful small sets of DoFs which can be collected and processed for object detection and characterization. We will concentrate on spatial/temporal/Doppler. Consider the inclusion of prior information to help in reducing the required number of DoFs for collection and exploitation.

$$\bar{E}^{r}(\bar{r};t)=\int_{V_c}\overline{\overline{G}}(\bar{r}-\bar{r}^{'})\bar{J}_{c}(\bar{r}')d\bar{r}'+\int_{V_t}\overline{\overline{G}}(\bar{r}-\bar{r}^{'}(t))\bar{J}_{t}(\bar{r}'(t))d\bar{r}'$$