DSO Mathematics: The Heart and Soul of the Far Side

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Teleprompter script

" As Yogi Berra pointed out, "It's tough to make predictions, especially about the future."

In 1900, Lord Kelvin addressed the British Association for the Advancement of Science. He asserted, *"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."*

How did the father of classical thermodynamics and one of the greatest physicists of the 1800's fail to anticipate general relativity, quantum mechanics, the electro-weak and strong forces, and the rest of 20th century physics? That same year, 1900, David Hilbert addressed the International Congress of Mathematicians on the future of **mathematics**. His words are as fresh today as they were then.

"Who among us would not be happy to lift the veil behind which is hidden the future; to gaze at the coming developments of our science and at the secrets of its development in the centuries to come?"

In his speech, Hilbert posed twenty-three problems. These twenty-three challenges drove much of mathematics over the next 100 years. Today, as we contemplate DARPA's approaching 50th anniversary, it is appropriate to examine the role that mathematics will play in our SECOND 50 years. Inspired by Hilbert, I feel like going out on a limb. You youngsters in the audience can come back and judge me in 50 years based on the challenges I foresee today. I've made my own list of twenty-three mathematical challenges. I see these as twenty-three opportunities to dramatically change mathematics and thereby strengthen the scientific and technological capabilities of DoD. I'm not going to go through all of them here—just come visit me at the DSO booth to pick up your own copy.

But I would like to talk to you today about how we will, with your help, dramatically advance the state of the art of mathematics at DARPA. Why do we bother? Because mathematics is the ultimate intellectual compression. Mathematics is often able to pack infinite amounts of wisdom into a single image or declarative statement. Furthermore, mathematics formalizes novel ways of thinking in order to extract deep consequences.

Some people see a great divide between applied and theoretical mathematics, with the latter regarded as a subject removed from the real world. I came to DARPA to change this perception.

In DSO, we want to invent new mathematics for new problems because traditional mathematical approaches are easily overwhelmed by the sheer scale of realistic problems, such as predicting weather. As a result, in DSO, mathematics is a synergy of theory and application in which the whole is greater than the sum of its parts.

It helps to remember that mathematicians traffic in patterns. Many brilliant Far Side ideas are inspired by seeing deep and unexpected structure where others perceive only chaos. Patterns can be qualitative, appearing as conservation laws or global geometric features. They can be quantitative, manifesting themselves in nature with incomparable precision...Or they can be a surprising union of the two. In 1900, Hilbert believed the best way to inspire future generations to uncover these patterns was to set forth great problems. Had Professor Hilbert been employed at DARPA at the time, I know he would have been pitching his challenges to Dr. Tether! I, too, think in patterns. So let me weave for you the tapestry of thought that runs through my challenges.

First, we must develop new mathematics to meet DARPA-hard problems. Traditionally, mathematics has not been productively applied to key challenges in biology, medicine, neuroscience, and the social sciences. These real world problems intrinsically possess many degrees of freedom. Conventional thinking, however, has resulted in inadequate mathematical formulations, which lack the power to extract crucial information. We need to discover new mathematics to accurately and reliably explain the essential high-dimensional nature and dynamics of these problems. These fundamentally better models must capture nature's stochasticity and uncertainty, something that Hilbert missed. Challenges here include developing a new mathematical theory to build a functional model of the brain that is mathematically consistent and predictive, rather than merely biologically inspired. Another challenge is finding new mathematics to replace the traditional partial differential equations approach to game theory and optimization. Advances here will illuminate social and economic phenomena, as well as predict the evolution of dynamic networks.

Second, let's build on past successes and develop new mathematics to address higher-level problems. Other Defense challenges, arising from physics, materials science, engineering, and computer science, seek to elevate mathematical successes of the past—to redefine the theoretical and computational frontiers. We need new theories of principled calculation, inspired by ever increasing computer power, to dramatically accelerate computational capabilities. These theories will yield new algorithms, methods, and tools, based in part, on duality, non-convex optimization, and numericalalgebraic-geometry—and the consequences will be *powerful*. Advances in computational topology and geometry will be used in novel ways to create pivotal breakthroughs in statistics and sensing applications. What are the challenges here? In the last century we learned how quantum phenomena shape our world. In the coming century we need to develop the mathematics required to control the quantum world. Similarly, classical fluid dynamics and the Navier-Stokes Equation were extraordinarily successful in obtaining quantitative understanding of shock waves, turbulence, and solutions. Now, new methods are needed to tackle complex fluids such as foams, suspensions, gels, and liquid crystals.

Finally, we must address the fundamental issue of scale. Today's collection techniques produce overwhelming mountains of data and information. Formidable counting tools at one scale are not useful at another. As Einstein observed, "Not everything that can be counted counts, and not everything that counts can be counted." Our challenges here include constructing models to handle complex dynamical systems with billions of degrees of freedom, such as those arising in climatology, biology, and the partial differential equations underlying non-equilibrium statistical mechanics. Developing the multi-scale mathematics required to be successful across scales and applications is perhaps the grandest challenge of all.

Before coming to DARPA, I was a professor of mathematics. I enjoyed catching my students off-guard on the first day of every course. They expected me to go to the board and start writing equations. Instead, I handed out copies of this timeless Peter Arno cartoon. I love hearing the Captain say, *"Couple of kooks, I guess... They carry on like that every time we pass here."*

As a teacher, I wanted my students to learn to think from the Far Side—and what better way to illustrate what can happen when someone is trapped by his own preconceptions? At first, the students don't realize that there is a prisoner in this cartoon, and the prisoner... is the ship's **captain**. He is confined by his lack of imagination, a dungeon far more limiting than any physical barrier. He will never be a DARPA performer.

My role at DARPA is to bring Far Side mathematics out of total, non-intuitive abstraction into the arena of basic research, where it can be applied to problems for the DoD. To do this, I have to think differently—and I need you to think differently, too, so you can help us solve these problems. Let me tell you about a couple of our programs that are already leaning into the future and how they will revolutionize DoD's technological capabilities:

The Far Side idea for our (TDA)—Topological Data Analysis program is that data has shape. TDA uses novel mathematical concepts and techniques to determine the intrinsic geometry of massive data sets, and then develops new tools to exploit that knowledge. In TDA, we have found shocking high-dimensional patterns in the statistics of natural images. This has led us to build novel, non-linear, compression schemes that will revolutionize the way images and movies are analyzed. These same techniques have uncovered dynamical patterns of neuron cooperation in stimulated firings in the V1 visual cortex. We are able to ask and begin to answer the question, *"How many neurons does it take to recognize a family of patterns?"* Even more surprising, we have uncovered unexpected and highly structured behavior when the firings are unevoked, and we have begun to extract temporal-spatial subtleties from the data.

These results demonstrate, for the first time, that computational topology offers novel tools to tackle fundamental questions about the way information is represented in the nervous system. We have also used these ideas to construct novel, non-linear, non-invasive medical statistics to assist doctors in understanding risks when assessing patients in intensive- and critical care situations. And we have built a new family of statistical tools to gain insight into massive metagenomics data sets in biology. These accomplishments all flow from a new theoretical framework, a calculus for data sets if you will, that will allow us to use these powerful techniques in many other settings. This is a glimpse of a profound new frontier in mathematics and its applications. Massive data sets are ubiquitous across all of DARPA's strategic thrusts, and I believe this technology will impact each and every one of them.

Our FAThM—Focused Areas in Theoretical Mathematics program started as a purely theoretical effort to investigate deep connections between number theory, geometry, harmonic analysis, and quantum physics. One might ask, *"Why would DARPA fund esoteric mathematics with no evident near-term benefit?"* Let me tell you why. In the coming months, seemingly esoteric ideas from this theoretical mathematics program will be used to design new materials with novel properties that have the long-range potential to produce nano-level computing devices. Not a bad bonus for a program that will also yield deeper insight into the fundamental symmetries of quantum theories of interactions between subatomic particles. FAThM is a powerful example of Arnold's truism that, *"the consequences of an investigation are many times more important than the original question ..."* As he reminds us, *"The initial goal of Columbus was to find a new way to India...The discovery of the New World was just a by-product."*

And, speaking of new worlds, our FunBio—Fundamental Laws of Biology program seeks to transform biology from a descriptive to a predictive science through development of new mathematics tailored to the most fundamental biological questions. Skeptics, like the captain in the cartoon, will say that problems in biology are too complex to solve mathematically. FunBio's Far Side quest is to discover new mathematics that work in biology with the **same power and effectiveness** that mathematics has traditionally wielded in physics and technology. Key FunBio insights include the discovery that there is a biological quantum theory for evolution that exhibits a wave-particle duality for the genome. You heard me correctly: imagine – wave-particle duality in biology! When biologists traditionally study genomes they treat those individual strands of DNA like particles. However, when a FunBio performer used quantum mechanical methods to model horizontal gene transfer in viruses, he discovered that they behaved like waves in exact analogy to the way instantons act in quantum mechanical tunneling. Thanks to this revelation, I believe mathematical biology will never be the same.

These programs reinforce Feynman's dictum that *"People who wish to analyze nature without using mathematics must settle for a reduced understanding."* I believe a new Golden Age of Mathematics is just beginning. Mathematics is the oldest of the sciences, yet it remains youthful, dynamic, and vibrant. It will always spawn Far Side ideas. And that is why Mathematics will always be the heart and soul of DSO—and, come to think of it, of DARPA itself!

Thank you. And now it is my great pleasure to introduce DSO's own wizard of software, Dan Kaufman.